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HANDLING NON-POSITIVE VARIABLES IN DATA ENVELOPMENT ANALYSIS

Marcin Krzysztof WALCZAK

Lodz University of Technology, Faculty of Organization and Management; marcin.walczak@dokt.p.lodz.pl, ORCID: 0000-0002-3351-1531

Purpose: The purpose of this paper is to identify and compare the main extensions of Data Envelopment Analysis (DEA) that enable efficiency measurement in the presence of non-positive input and output data. The study aims to examine how the choice of method affects efficiency classification and ranking, addressing a gap in the literature where existing contributions are dispersed and method-specific.
Design/methodology/approach: The study follows a mixed-method research design. A systematic literature review was conducted using the Scopus database to identify DEA models capable of handling negative data. This was supplemented by semi-structured interviews with five academic experts specializing in DEA to validate the relevance and practical use of the identified approaches. A simulation experiment was then performed using a synthetic dataset of 80 decision-making units representing banks, containing both positive and negative variables. Four widely applied models were compared: the Range Directional Model, the Modified Slacks-Based Measure, the Semi-Oriented Radial Measure, and a Two-Stage DEA model.
Findings: The results indicate that efficiency classification is strongly dependent on the selected DEA model. The number of units identified as efficient differs substantially across methods, with the Two-Stage DEA producing the strictest efficiency frontier and the Range Directional Model the broadest. At the same time, rank-order correlations across models remain consistently high, suggesting that relative performance rankings are largely preserved despite differences in frontier definition.
Research limitations/implications: The study is limited by the use of a synthetic dataset and a small number of expert interviews, which may constrain external validity. Future research should apply the compared models to empirical datasets from different sectors and conduct robustness and sensitivity analyses.
Practical implications: The findings emphasize the need for explicit, purpose-driven model selection in DEA studies involving negative data, as methodological choices directly influence efficiency benchmarks and managerial conclusions.
Social implications: By increasing transparency in efficiency measurement, the study may support more informed decision-making in areas such as banking and public administration, contributing to improved resource allocation.
Originality/value: The paper offers a comprehensive synthesis and direct comparison of major DEA models designed to handle non-positive data, demonstrating that efficiency is a model-dependent construct and providing guidance for researchers and practitioners.

Keywords: Data Envelopment Analysis; negative data; efficiency measurement; benchmarking; banking sector.
Category of the paper: Research Paper.

1. Introduction

The concept of efficiency, alongside notions such as performance and effectiveness, has always stood at the core of business studies and organizational research. The etymology of the term *efficiency* traces back to the Latin word *efficio*, which conveys meanings such as to *perform*, *complete*, *execute*, or *produce* (Rybicki, 2005). Conversely, according to the ISO 9000 standard, efficiency is defined as the ratio between achieved results and utilized resources. Bielski (1992) offers a particularly noteworthy conceptualization of efficiency, distinguishing between two perspectives. The deliberate approach focuses on predefined organizational goals, emphasizing decision-making that minimizes resource utilization. Conversely, the systematic approach underscores the importance of the organizational environment and the capacity to acquire the necessary resources from it Bielski (1992). However, Drucker (1963) provided the simplest yet meaningful definition: *efficiency is concerned with doing things right*.

In recent years, there has been a growing academic interest in utilizing the Data Envelopment Analysis (DEA) method to assess the efficiency of various economic systems. Conventional Data Envelopment Analysis (*DEA*) models have been developed based on the assumption that all inputs and outputs are non-negative (Maryam, Rostamy-Malkhalifeh, 2015). This method's growing adoption and popularity has highlighted a critical need to address the challenge posed by negative data, a common phenomenon in practical, real-world scenarios (Maryam, Rostamy-Malkhalifeh, 2015). DEA is a non-parametric methodology that employs linear programming techniques to evaluate the efficiency of units called Decision Making Units (*DMUs*). This approach incorporates multiple inputs and outputs across various units to determine relative technical efficiency. Within the DEA framework, efficiency is defined as the ratio of the weighted sum of outputs to inputs. A key attribute of DEA lies in its flexible and adaptable structure, which is grounded in the data available for analysis (Xie et al., 2021). The origins of the method can be traced back to Farrell's foundational work (Farrell, 1957), initially applied within the agricultural sector. The term Data Envelopment Analysis was subsequently introduced by Charnes et al. (1978), with further developments contributed by Banker et al. (1984). Efficiency in DEA is calculated as the ratio of weighted outputs to weighted inputs, producing values that range between zero and one (Walczak, Marcinkiewicz, 2022). The efficiency ranking of units is established by computing these ratios across all units in the dataset (Andersen, Petersen, 1993). A unit is deemed efficient if its efficiency ratio reaches the efficiency frontier, represented by a value of one. Units with ratios below one are classified as inefficient (Antunes et al., 2021). In essence, DEA facilitates the distinction between efficient and inefficient units by evaluating their input utilization relative to output production. Additionally, DEA integrates the concept of benchmarking, as highlighted by Ruiz and Sirvent (2019), who stated, *Using DEA for benchmarking ensures an evaluation in terms of targets that are both attainable and represent best practices*.

Two basic models of Data Envelopment Analysis (*DEA*) can be identified: the CCR model and the BCC model. The CCR model, developed by Charnes *et al.* (1978), is based on the assumption of constant returns to scale (*CRS*) (Gizaw, 2019). In contrast, the BCC model, introduced by Banker et al. (1984), serves as an extension of the CCR model. This model operates under the assumption of variable returns to scale (*VRS*) (Gizaw, 2019). The presence of negative data in Data Envelopment Analysis poses a fundamental methodological problem because basic DEA models are constructed on the assumption that all input and output values are nonnegative (Charnes et al., 1978). When variables take on negative values, the input–output ratios that form the basis of DEA efficiency measurement become mathematically undefined, leading to infeasibility or meaningless efficiency scores (Portela et al., 2004). Moreover, the core axioms of DEA, including free disposability and monotonicity, are violated when data include negative observations, since increasing a negative output or decreasing a negative input may not correspond to a genuine improvement in performance (Cooper et al., 1999)

This research involves a comparative review of previous studies focusing on methodologies for managing negative data within DEA frameworks, with particular attention to their implications for efficiency measurement and practical application.

Based on the initial literature review the following research questions were posed:

Q1. What are methods used for dealing with negative data while implementing DEA method?

The literature review revealed that, although numerous Data Envelopment Analysis (DEA) models have been developed to address the problem of non-positive variables, there is still a lack of comprehensive publications that synthesize these contributions. Existing studies tend to concentrate on developing new model variations or applying them within specific sectors. This study contributes to the literature in several ways. First, it provides a structured synthesis of the main DEA extensions designed to handle non-positive data, focusing on models that are most frequently applied in practice rather than on isolated methodological proposals. Second, it offers a direct, head-to-head comparison of these models using a common dataset, which allows differences in efficiency frontiers and rankings to be attributed solely to methodological choices. Third, by combining a systematic literature review with expert interviews and a controlled simulation experiment, the study links theoretical model properties with their practical implications for efficiency assessment.

Q2. How does the implementation of particular methods of dealing with non-positive values influence the final result of Data Envelopment Analysis?

The article seeks to conduct a comparative analysis of results obtained through selected models, highlighting their relative effectiveness and practical utility. In this way, the study not only fills an important gap in the literature but also offers readers a structured overview of available approaches and practical guidance on how selecting a certain method influences on the results.

2. Literature Review

The Scopus database was selected for the research, using the following search terms: “DEA” OR “Data Envelopment Analysis” AND “negative” OR “undesired”. The search was restricted to abstracts containing the specified keywords. This initial search yielded 2055 documents. Further refinement of the selection was performed and described in Table 1.

Table 1.
Literature Review: Refinement of the selection

Criterium	No of papers fulfilling
Search terms on Scopus: “DEA” OR “Data Envelopment Analysis” AND “negative” OR “undesired”	2005
Only articles and book chapters	1987
Only final versions	1856
Only documents in English	1691
Collocation of word “ negative” or “undesired” with “data”	98

These constraints resulted in a reduction of the available documents. No geographical constraints were introduced. Each article's abstract was then reviewed to assess how the terms *negative* and *undesired* were applied. It was found that the database contained articles where the term *negative* was frequently associated with words such as *influence*, *effect*, and *ratio*. However, in the context of the present study, the terms *negative* and *undesired* were specifically intended to describe *data*, *variable(s), input(s), or output(s)*. Notably, 98 articles employed the term *negative* about *data*. Surprisingly, no articles that used the term *undesired* in conjunction with *data* were found.

To avoid relying solely on database inclusion, an additional journal quality screening was conducted. Only journals indexed in Scopus or in the Web of Science Core Collection were retained. Journals classified in Q1–Q3 according to the Scimago Journal Rank were considered acceptable, while Q4 journals and titles without quartile assignment were excluded. To further reduce the inclusion of marginal outlets, a minimum impact threshold was applied: Journal Impact Factor (JCR) of at least 1,0 or CiteScore of at least 1,0. 54 articles published in journals not meeting these criteria were removed from the final sample.

After examining the references in the retrieved articles and reviewing the citations, it was determined that some important papers in the field were absent from the final selection. Consequently, the author decided to include an additional eight papers, following a thorough reevaluation of their quality and the reliability of the journals which they were published in. The backward snowballing method was incorporated. A subsequent review was conducted based on the 54 articles that met these criteria.

Ali and Seiford (1990) were among the first researchers to address the issue of handling non-positive data in Data Envelopment Analysis (DEA), focusing on the presence of zero values. They demonstrated that affine displacement of undesirable data does not impact the efficiency frontier; however, in the context of the BCC model, such displacement can influence the efficiency scores of inefficient units (Ali, Seiford, 1990). This work was further advanced by Pastor (1994), who proposed increasing the values of undesirable variables by adding a sufficiently large positive constant, as referenced in the work of Portela *et al.* (2004). Pastor's approach was subsequently adopted by Seiford and Zhu (2002) and Bowlin (1998). Building on this foundation, Lovell and Pastor (1995) developed a DEA variant that employs a weighted sum of slacks, where the weights correspond to the inverse of the standard deviations of each input and output associated with the slack. This methodology was later extended by Pastor (1996), and incorporated by Thrall (1996), Tone et al. (2020), as well as Cheng et al. (1999). For cases where inputs or outputs are entirely non-positive, Scheel (2001) proposed treating these variables as positive outputs or inputs. This approach was subsequently employed by other researchers, including Cheng et al. (2013), and Zhu (2003). Halme et al. (2002) noted that Zhu, in an unpublished manuscript from 1994, suggested a complex method involving data translation and the application of controlled envelopment analysis concepts. Halme et al. (2002) further proposed replacing interval scale variables with corresponding pairs of ratio scale variables, provided the derived variable is computed as the difference between two ratio-scale variables. This method was later adopted by Pastor and Ruiz (2007), Dehnokhalaji et al. (2010), and Mohammadpour et al. (2015). Portela et al. (2004) introduced the Range Directional Model (*RDM*), which handles non-positive variables without requiring data transformation. It illustrates the concept of variable returns to scale, and it is not possible to integrate alternative returns to scale assumptions within this framework. This method, grounded in the directional distance function, was further developed by Pastor and Ruiz (2005) and adopted by researchers such as Asmild and Pastor (2010), Peykani et al. (2022), Yousefi et al. (2019), Tavana et al. (2018), as well as Banihashemi and Navidi (2017). Sharp et al. (2007) proposed the Modified Slack-Based Measure (*MSBM*), which accommodates both positive and negative data. This model addresses the absence of translation invariance found in the slacks-based measure model by incorporating concepts from the range directional model (RDM). By considering individual input and output slacks, the MSBM model offers a more accurate assessment of inefficient units. Consequently, it typically results in lower efficiency scores for inefficient DMUs compared to the RDM (Sharp et al. (2007). This method has been utilized by other researchers, including Cui, et al., (2022), Wang et al. (2005), and Lee et al. (2017). Another prominent approach for handling negative data involves partitioning positive and negative variables. Emrouznejad et al. (2010) applied this concept to propose the Semi-Oriented Radial Measure (*SORM*) for efficiency evaluation, which was later refined by Kazemi et al. (2014), as well as Kaffash et al. (2018). It transforms each input-output variable fundamentally as the sum of two components: one representing its negative value and the other its positive value. This representation offers the benefit of allowing the negative aspect of a variable to be addressed in absolute terms, thereby presenting it in a positive format without the need for arbitrary adjustments to the origin that might otherwise be required to obtain positive values (Matin et al. 2014). This method has been widely adopted by researchers, including Mo et al. (2020), Emrouznejad et al. (2010), Kenjegalieva et al. (2011), Toloo et al. (2015). Kenjegalieva et al. (2011), and Kumari et al. (2024). Cheng et al. (2013) introduced a modified version of the traditional radial DEA model, replacing original input and output values with their absolute values. This method was subsequently employed by Tung et al. (2018). Izadikhah and Saen (2016) proposed a two-stage DEA model where each unit is structured as two interconnected sub-units, with the outputs of the first stage serving as inputs for the second stage. This two-stage framework operates in the context of undesirable data, where the initial stage generates the final outputs of the network, which subsequently serve as inputs for the second stage. This innovative approach has been applied by researchers such as Mukherjee et al. (2024), Hsu et al. Chiu (2023), and Singpai and Wu (2021). Finally, Lin and Chen (2017) introduced a super-efficiency model based on the directional distance function, which was later expanded upon by Lin and Liu (2019).

Based on the literature review, it can be concluded that the two-stage DEA model introduced by Izadikhah and Saen (2016), the semi-oriented radial measure (SORM) developed by Emrouznejad et al. (2010), the range directional model (RDM) proposed by Portela et al. (2004), and the Modified Slack-Based Measure (MSBM) introduced by Sharp et al. (2007) are among the most widely adopted methods. This selection is corroborated by other researchers, such as Kaffash et al. (2018), although they did not mention the two-stage DEA model. This omission is likely attributable to the fact that the first paper on this model was published in 2016, while their selection was made in 2018, and the two-year period may not have been sufficient for the new methodology to gain attraction among scholars. In general, methodologies developed post-2000 have been frequently utilized in a substantial number of studies.

3. Methodology

3.1. Extended models of Data Envelopment Analysis

As mentioned earlier, four models have been selected for the comparative analysis. This section provides their technical overview, after which the models will be tested on datasets containing negative values to examine their impact on efficiency results.

The Range Directional Model is grounded in the Directional Distance Function (DDF) methodology (for VRS) (Chambers et al., 1996, 1998), which measures inefficiency by expanding outputs and contracting inputs along a chosen direction:

$$max = \beta_o \tag{1}$$

Subject to:

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta_o g_{yr}, r = 1, \dots, s, \tag{2}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta_o g_{xi}, i = 1, \dots, m, \tag{3}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, g_{xi} \geq 0, g_{yr} \geq 0 \tag{4}$$

The model mentioned above was further modified by Portela et al. (2004) to address cases where some values are negative, so that it always produces feasible improvement directions. They have also introduced an ideal point - defined by the maximum observed outputs and the minimum observed inputs across all units. Based on this reference, two vectors were determined (5) (6), which represent the potential range of improvement for unit o.

$$R_{ro} = \underset{J}{max}\{y_{rj}\} - y_{ro}, r = 1, \dots, s \tag{5}$$

$$R_{io} = x_{io} - \underset{J}{min}\{x_{ij}\}, i = 1, \dots, m \tag{6}$$

Build on the (5) and (6), the Range Directional Model was introduced.

$$max = \beta_o \tag{7}$$

Subject to:

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \beta_o R_{ro}, r = 1, \dots, s, \tag{8}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \beta_o R_{io}, i = 1, \dots, m, \tag{9}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \tag{10}$$

The Range Directional Model was originally formulated as a single-stage approach, where a decision-making unit is treated as a "*black box*" that transforms inputs into outputs. Building upon this concept, Tavana *et al.* (2018) incorporated the model into a two-stage DEA, in which the internal processes of each DMU are explicitly considered.

In the two-stage DEA framework, each DMU is decomposed into two sub-processes:

- Stage 1 (S1): transforms initial inputs x_{ij} into intermediate products z_{dj}.
- Stage 2 (S2): consumes the intermediate products z_{dj} to generate the final outputs y_{rj}.

Accordingly, the overall efficiency of a DMU is determined as a composition of the efficiencies of the S1 and S2.

The two-stage DEA is constructed by introducing two ideal points, separately for S1 and S2:

$$I' = \left(\max_j z_{dj}, \min_j x_{ij}\right), I'' = \left(\max_j y_{rj}, \min_j z_{dj}\right) \tag{11}$$

Based on these reference points, two improvement direction vectors (11) are defined, representing the ranges of possible improvement for each stage.

The objective function of the two-stage RDM is specified as:

$$\max w_1\theta_1 + w_2\theta_2 \tag{12}$$

where θ_1 *and* $\theta_2$represent the improvement parameters for S1 and S2, respectively, while w_1 *and* $w_2$are the importance weights assigned to each stage ($w_1 + w_2 = 1$). In this analysis $w_1 = w_2 = 0{,}5$.

The above is subject to:

At the S1:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip} - \theta_1 R_{x_i}^1, i = 1, \dots, m, \tag{13}$$

$$\sum_{j=1}^{n} \lambda_j z_{dj} \geq z_{dp} - \theta_2 R_{z_d}^1, d = 1, \dots, D, \tag{14}$$

At the S2:

$$\sum_{j=1}^{n} \mu_j z_{dj} \leq z_{dp} - \theta_2 R_{z_d}^2, d = 1, \dots, D, \tag{15}$$

$$\sum_{j=1}^{n} \mu_j y_{rj} \geq y_{rp} - \theta_2 R_{y_r}^2, r = 1, \dots, s, \tag{16}$$

Additional convexity conditions apply:

$$\sum_{j=1}^{n} \lambda_j = 1, \sum_{j=1}^{n} \mu_j = 1, \lambda_j, \mu_j \geq 0 \tag{17}$$

Linkage constraint:

$$\sum_{j=1}^{n} \lambda_j z_{dj} = \sum_{j=1}^{n} \mu_j z_{dj}, d = 1, \dots, D, \tag{18}$$

Efficiency scores are thus expressed as ($\rho_1$at the stage 1, ρ_2 at the stage 2, ρ as overall efficiency)

$$\rho_1 = \theta_1^*, \rho_2 = \theta_2^*, \rho = w_1\rho_1 + w_2\rho_2 = 1 - (w_1\theta_1^* + w_2\theta_2^*) \tag{19}$$

Building on the vectors R_{ro}, and R_{io}, suggest by Portela *at al.* (2004) Sharp *et al.* (2005) introduced the Modified Slacks-Based Measure (MSBM), which extends classic Tone's (2001) SBM model. By incorporating the SP Range into the normalization of slacks, the MSBM ensures both unit-invariance and translation-invariance. The model is formulated as:

$$\rho = \min \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{P_i^-}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_i^+}{P_r^+}} \tag{20}$$

subject to standard DEA feasibility constraints, where $s_i^-$and s_i^+ denote input and output slacks, respectively. The efficiency score ρ lies in the range $[0,1]$, and equals 1 only if all slacks are zero.

Last, but not least Semi-Oriented Radial Measure (SORM) suggested by Emrouznejad et al. (2010). Since the negative values in the data may occur two index sets are defined:

$$I = \{i : x_{ij} \geq 0 \; \forall j\}, L = \{l : \exists j, x_{lj} < 0\} \tag{21}$$

$$R = \{r : y_{rj} \geq 0 \; \forall j\}, K = \{k : \exists j, y_{kj} < 0\} \tag{22}$$

Later, the negative data is decomposed into non-negative parts.

$$x_{lj} = x_{lj}^1 - x_{lj}^2, x_{lj}^1 = \max(x_{lj}, 0), \; x_{lj}^2 = \max(-x_{lj}, 0) \tag{23}$$

$$y_{kj} = y_{kj}^1 - y_{kj}^2, y_{kj}^1 = \max(y_{kj}, 0), \; y_{yj}^2 = \max(-y_{kj}, 0) \tag{24}$$

While the two stage DEA, SORM, and RDM are directionals models, which means there is not clear input or output orientation, the MSBM explicitly evaluates inefficiency through input and output slacks, thus allowing for a more direct interpretation in terms of input reduction and/or output expansion.

For the evaluation of a DMU j_0 the input-oriented model is formulated as:

$$minh \tag{25}$$

Subject to:

$$\textstyle\sum_{j=1}^{n} x_{ij}\lambda_j - hx_{ij_0} \leq 0 \;\; \forall i \in I, \tag{26}$$

$$\textstyle\sum_{j=1}^{n} x_{lj}^1\lambda_j - hx_{lj_o}^1 \leq 0 \;\; \sum_{j=1}^{n} x_{lj}^2\lambda_j - hx_{lj_o}^2 \geq 0 \;\; \forall l \in L, \tag{27}$$

$$\textstyle\sum_{j=1}^{n} y_{rj}\lambda_j \geq y_{rj_0} \;\; \forall r \in R, \tag{28}$$

$$\textstyle\sum_{j=1}^{n} y_{kj}^1\lambda_j \geq y_{kj_o}^1, \; \sum_{j=1}^{n} y_{kj}^2\lambda_j \leq y_{kj_o}^2 \;\; \forall k \in K, \tag{29}$$

$$\textstyle\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 1 \tag{30}$$

Analogously, the output-oriented version is expressed as:

$$min\varphi \tag{31}$$

$$\textstyle\sum_{j=1}^{n} x_{ij}\lambda_j \;\leq x_{ij_0} \;\; \forall i \in I, \tag{32}$$

$$\textstyle\sum_{j=1}^{n} x_{lj}^1\lambda_j \leq x_{lj_o}^1, \; \sum_{j=1}^{n} x_{lj}^2\lambda_j \geq hx_{lj_o}^2 \;\; \forall l \in L, \tag{33}$$

$$\textstyle\sum_{j=1}^{n} y_{rj}\lambda_j - \varphi y_{rj_0} \geq 0 \; \forall r \in R, \tag{34}$$

$$\textstyle\sum_{j=1}^{n} y_{kj}^1\lambda_j - \varphi y_{kj_0}^1 \geq 0, \; \sum_{j=1}^{n} y_{kj}^2\lambda_j - \varphi y_{kj_0}^2 \leq 0, \; \forall k \in K, \tag{35}$$

$$\textstyle\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \tag{36}$$

For practical applications, it is necessary to ensure the models are bounded. The following conditions hold:

- The input-oriented model is bounded if and only if:

$$d_{j_0} = max\left\{\max_{i \in I} x_{ij_0}, \max_{l \in l} x_{lj_0}^1\right\} > 0 \tag{37}$$

- The output-oriented model is bounded if and only if:

$$b_{j_0} = max\left\{\max_{r \in R} y_{rj_0}, \max_{k \in K} y_{kj_0}^1\right\} > 0 \tag{38}$$

- The output-oriented model is bounded if and only if:

$$a_{j_0} = min(d_{j_0}, b_{j_0}) > 0 \tag{39}$$

3.2. Supplementary findings test

To validate the findings of the literature review, a qualitative research, based on semi-structured interviews with five academic experts specializing in Data Envelopment Analysis (DEA) was deployed. The interview guide was designed following a systematic framework. Kallio et al. (2016) proposed a five-step process for developing semi-structured interviews, which includes defining the aim, reviewing the literature, drafting questions, piloting, and refining. This study followed that model to ensure validity and reliability. The final guide covered five thematic blocks (Castillo-Montoya, 2016): (1) general background and DEA experience, (2) challenges with negative variables, (3) methodological preferences, (4) practical applications, and (5) final remarks. The interviews were conducted via online conferencing platforms, each lasting approximately 45 minutes. Sessions were not recorded, yet instant transcription was turned on. While the participants' backgrounds ranged from finance and public administration to logistics or banking, there was strong convergence in their responses: negative data is both a recurrent phenomenon and a methodological challenge in DEA, and its treatment directly influences efficiency scores, rankings, and benchmarks. Most of the experts (experts A, C, E) emphasized the Range Directional Model. They described it as particularly effective in complex network settings, because directional goals can be aligned with policy or managerial priorities. MSBM was widely highlighted (experts A, C, D) as a method that provides conservative efficiency estimates by penalizing unaddressed slacks. Experts explained that MSBM avoids the inflation of efficiency scores that may occur in radial models when negative values are present. This method was consistently valued as a diagnostic tool that goes beyond ranking to show where inefficiencies reside, making it attractive for regulators and managers seeking actionable improvements. Two experts (Expert B and D) reported favorably on SORM, while three experts (Expert B, C, and E) highlighted the advantages of two-stage DEA formulations. The consensus was that multi-stage models not only allow the inclusion of negative values but also provide more detailed guidance for targeted improvements.

Participants' answers undergo thematic and qualitative analysis, which resulted in following statements:

1. Issue with applicability of non-positive data in Data Envelopment Analysis is common, classic models are unable to deal with such variables, yet while analysis real world problems such figures cannot be omitted, which in turn lead to the need of variable or DMU exclusion.
2. List of extended/modified Data Envelopment Analysis methods yielded based on both, the literature review and semi-structured interviews is identical.

The author considered limitations to ensure that all necessary measures were taken to mitigate the negative influences caused by potential limitations. For studies with multiple participants, the workload can delay reporting and reduce efficiency. Thus, a group of 5 experts was selected. The author, based on the initial review, chose only these researchers whose portfolios are characterized by a high volume of papers involving Data Envelopment Analysis. Taking into consideration that popularity of Data Envelopment Analysis may differ across different countries, experts from 3 countries (4 universities) were included in analysis. Although some of the limitations were possible to be mitigate, others should be taken into consideration while interpreting the article's findings.

3.3. Simulation experiment design

To evaluate and compare the performance of different DEA extensions designed to handle negative data, a synthetic dataset was constructed. The dataset comprises 80 decision-making units (DMUs), each representing a hypothetical banking institution. For each DMU, seven inputs and three outputs were generated, reflecting variables that are commonly used in efficiency studies of the banking sector. Examples of the input variables include staff (measured as full-time equivalents), the number of branches and ATMs, operating expense changes, cost of funds, risk-weighted assets, and changes in the ratio of non-performing loans. The outputs reflect financial and business performance: net interest income, non-interest income, and loan growth rates.

Because the aim of this study is methodological rather than empirical, the dataset was not drawn from banking records but generated artificially. To ensure that the data resemble realistic banking sector values, while still allowing for the presence of both positive and negative figures, the GPT-5 large language model was employed (developed by OpenAI) to design and generate the data. The model was instructed to create variable distributions reflecting the economic and financial logic of the sector. Randomness was introduced via normally distributed draws, clipped to maintain plausible ranges (operating expense changes between –30% and +40%, loan growth between –40% and +60%, cost of funds between –200 and 600 basis points).

The bounds for operating expense changes were based on empirical evidence from the banking and merger literature. Reported post-merger cost savings in noninterest or operating expenses typically range between 15% and 40%, with documented declines of around 30% within the first year (DeYoung, 1993; Board of Governors of the Federal Reserve System, 1998). These findings support the use of –30% as a conservative lower bound. At the same time, institutional evidence shows that operating expenses may temporarily increase due to restructuring and integration costs, with year-on-year increases of up to 40% reported in practice (Central Bank of the Republic of Turkey, 2020). Together, these studies justify the – 30% to +40% interval as a plausible range for operating expense changes. While Dang (2019) observed the yearly growth of banks ranges from over 90% to – 45%, Godlewski and Weill (2018) based on the data from banks belonging to the Bank Polskiej Spółdzielczości indicates that in such specific case the growth is lower from around 50% to -15%. These discrepancies highlight cross-country and sample-specific heterogeneity, which motivated the use of intermediate values as conservative bounds for the upper and lower limits in the simulation. Regarding bank saving costs, the empirical evidence shows that they may temporarily decline by more than 150 basis points relative to pre-crisis benchmarks (Reserve Bank of New Zealand, 2024), justifying a –200 bps lower bound. Conversely, periods of monetary tightening and financial stress have been associated with increases in funding costs of several hundred basis points, with documented cases reaching approximately 600 basis points (European Central Bank, 2024; Powell, 2002).

The resulting dataset was exported into Excel format with two sheets. The first sheet contains the main data matrix (80 × 11), while the second provides a codebook explaining the role, unit of measurement, and interpretation of each variable.

4. Differences and Similarities among selected methods

All four models, mentioned in previous section, were developed to extend Data Envelopment Analysis to cases where negative inputs and/or outputs are present. Despite this common motivation, they differ in their theoretical foundations, mathematical formulations, and the way efficiency is computed and interpreted.

A key similarity is that they preserve the enveloping principle of DEA: efficiency is always evaluated relative to a piecewise linear production frontier constructed from observed decision-making units. Moreover, RDM, Two-stage DEA, and SORM are directional or radial models. They rely on a single proportional factor that scales inputs and outputs simultaneously along a chosen direction (Emrouznejad et al., 2010) MSBM, by contrast, belongs to the slack-based family. This method instead of relying on a single radial adjustment, it uses input and output slacks, which measure non-proportional inefficiency in each variable (Tone et al., 2020).

The main differences arise from the structure and treatment of inefficiency. RDM and SORM treat each DMU as a “black box,” while Two-stage DEA explicitly decomposes the production process into two sub-stages (Lychev et al., 2023). This allows the Two-stage DEA to provide stage-specific efficiency scores in addition to the overall measure. In terms of inefficiency measurement, RDM and Two-stage DEA calculate proportional improvements with a single radial factor for each stage (Tavana et al., 2018), while SORM does the same after decomposing negative variables into positive and negative parts (Emrouznejad, Anouze et al., 2010). MSBM, on the other hand, captures inefficiency in a non-radial way, allowing each input and output to adjust differently, with efficiency expressed as a function of the average normalized slacks (Sharp et al., 2007). The strategies for handling negative data are also distinct. RDM introduces an “ideal point” composed of the maximum observed outputs and minimum observed inputs, and defines feasible improvement ranges relative to that point (Portela et al., 2004). Two-stage RDM applies the same logic separately for each stage of the process (Tavana et al., 2018). MSBM normalizes slacks by feasible ranges, thereby ensuring unit invariance and translation invariance (Sharp et al., 2007). SORM avoids both translation and normalization by splitting each negative variable into its positive and negative parts, and then applying a radial contraction or expansion factor (Cheng et al., 2013).

Another crucial difference is boundedness: while RDM and MSBM rely on feasibility conditions, SORM requires that the models remain bounded. Specifically, the input-oriented version is bounded if at least one input is strictly positive, the output-oriented version is bounded if at least one output is strictly positive, and both are bounded simultaneously if the minimum of these two values is greater than zero. In the Two-stage RDM, boundedness depends on the feasibility of both stage-specific models and the linkage constraint.

Interpretability also varies across models. RDM offers an intuitive interpretation as a proportional "push" toward the ideal point, though results depend on the chosen range direction (Portela et al., 2004). Two-stage RDM extends this by breaking down overall inefficiency into stage-specific components, which provides diagnostic value (Tavana et al., 2018). MSBM is highly interpretable in terms of slacks: the efficiency score can be directly linked to the proportion of inputs to be reduced and outputs to be increased (Zarrn, 2023). SORM preserves the familiar feel of classical DEA, through input- or output-orientation, while extending it to cases where data may take negative values.

5. Research results

The comparative application of four DEA models (MSBM, SORM input- and output-oriented, Two-Stage DEA, and RDM) to the same dataset of 80 DMUs highlights differences in the classification of efficiency. The results are summarized in Table 2.

Table 2.
Efficient vs. inefficient DMUs across methods. Own elaboration based on own studies

Method	Number of DMUs	Efficient	Inefficient
MSBM	80	35	45
SORM (Input)	80	42	38
SORM (Output)	80	42	38
Two-Stage DEA	80	25	55
RDM	80	44	36

RDM identifies 44 efficient DMUs and 36 inefficient DMUs. Both the input- and output-oriented versions of SORM identify 42 efficient and 38 inefficient DMUs. MSBM identifies 35 efficient and 45 inefficient DMUs. The Two-Stage DEA model identifies 25 efficient DMUs and 55 inefficient DMUs.

To compare the overlap of efficient sets across methods, the Jaccard similarity index (Fletcher, Islam, 2018) was applied. This measure calculates the size of the intersection of efficient sets between two models divided by the size of their union.

Table 3.
Jaccard similarity between efficient sets. Own elaboration based on own studies

	MSBM	SORM_in	SORM_out	Two-Stage
MSBM	1.000	0.833	0.833	0.500
SORM_in	0.833	1.000	1.000	0.489
SORM_out	0.833	1.000	1.000	0.489
Two-Stage	0.500	0.489	0.489	1.000

Table 3 presents the Jaccard similarities among MSBM, SORM (input- and output-oriented), and Two-Stage DEA. The Jaccard index between MSBM and SORM equals 0.833 for both orientations. The similarity between MSBM and Two-Stage DEA equals 0.500. The overlap between SORM and Two-Stage DEA equals 0.489 for both orientations. The input- and output-oriented versions of SORM yield identical similarity values.

Rank-order consistency was examined using Spearman rank correlation coefficients to assess the similarity of performance rankings across models.

Table 4.
Spearman correlations of DMU rankings. Own elaboration based on own studies

	MSBM	SORM_in	SORM_out	Two-Stage
MSBM	1.000	0.938	0.938	0.990
SORM_in	0.938	1.000	0.989	0.925
SORM_out	0.938	0.989	1.000	0.925
Two-Stage	0.990	0.925	0.925	1.000

The rank correlations are uniformly high, all above 0.90, indicating that relative performance orderings are consistent across models. The highest correlation is observed between MSBM and Two-Stage DEA, showing that despite the latter's stricter efficiency criteria, the overall ranking of units remains nearly identical. Similarly, the near-perfect correlation between the two SORM variants confirms that orientation has no practical effect on rankings in this dataset. Preliminary checks for RDM suggest correlations in the range of 0.90 to 0.93 with MSBM and SORM, reinforcing the view that directional measures shift the efficiency threshold but not the relative hierarchy.

6. Discussion

The results demonstrate that efficiency classification in Data Envelopment Analysis is sensitive to the choice of model when non-positive data are present. Although all models were applied to the same dataset using identical inputs and outputs, the number of decision-making units classified as efficient varies substantially across methods. This finding highlights that efficiency, in such settings, is not an intrinsic property of the data alone but depends on the methodological framework used to evaluate it.

The observed differences across models reflect how inefficiency and non-positive values are treated within each DEA extension. Some approaches identify a relatively larger set of efficient units, whereas others impose more restrictive conditions, resulting in smaller efficiency frontiers. These differences indicate that efficiency frontiers are model-dependent rather than invariant. At the same time, the high Spearman rank correlations observed across all model pairs show that relative performance rankings remain largely stable. This suggests that while models differ in where the efficiency cutoff is placed, they tend to order decision-making units in a similar manner. The identical results obtained for the input- and output-oriented versions of the Semi-Oriented Radial Measure (SORM) indicate that orientation has no practical effect on rankings in the analyzed dataset. Furthermore, the substantial overlap between the efficient sets identified by MSBM and SORM, contrasted with the lower overlap involving the Two-Stage DEA model, points to structural differences in how these approaches define and constrain efficiency. Importantly, these differences should not be interpreted as inconsistencies in the results, but rather as natural consequences of distinct assumptions embedded in each model.

These findings extend the existing literature on DEA with non-positive data. Previous studies have primarily focused on proposing individual model extensions or demonstrating their applicability in specific empirical contexts (e.g., Portela et al., 2004; Sharp et al., 2007; Emrouznejad et al., 2010). While this body of work established that negative data can be accommodated within DEA, it offered limited insight into how different extensions compare when applied under identical conditions. The present study contributes by providing a direct, head-to-head comparison of widely used DEA models on the same dataset, thereby isolating the effect of methodological design choices on efficiency classification and ranking.

The results are broadly consistent with earlier findings suggesting that DEA rankings are relatively robust to changes in model specification (Andersen, Petersen, 1993). At the same time, the variation in the number of efficient units aligns with prior observations that alternative treatments of undesirable or non-positive data lead to shifts in the efficiency frontier (Portela et al., 2004; Sharp et al., 2007). What is shown more explicitly in this study is that these two phenomena coexist: substantial differences in frontier membership may occur alongside highly stable rankings. This distinction between ranking stability and frontier sensitivity has received limited explicit attention in previous research.

Taken together, the results confirm the underlying assumption that methodological choices in DEA models handling non-positive data materially affect efficiency outcomes. In particular, the analysis demonstrates that model selection influences which units are labeled efficient, while exerting a more limited influence on relative performance ordering. From a practical perspective, this underscores the importance of explicitly justifying model choice and of complementing point estimates with sensitivity analyses when DEA is applied in the presence of non-positive data.

Future research could extend this comparative framework to empirical datasets from different sectors, examine the sensitivity of results to alternative directional vectors, orientations, or stage weights, and explore hybrid approaches that combine desirable properties of the analyzed models. Such extensions would further clarify the practical implications of model selection and contribute to more transparent and robust applications of DEA in real-world settings.

7. Conclusions

This study set out to identify the main approaches of Data Envelopment Analysis that can handle non-positive data and to show how choosing among them changes measured efficiency as well as rankings. A scan of the literature, combined with expert perspectives, points to four models that are widely used. Although all preserve the general logic of Data Envelopment Analysis, they differ in how they treat translation invariance, the role of directions, the decomposition of processes, and whether inefficiency is captured proportionally or through variable-specific slacks.

When these methods are applied to the same dataset, the set of units labeled efficient changes across models, while the overall ranking of performance remains largely similar. This finding directly confirms that model choice affects efficiency classification even under identical data conditions. In practical terms, the selection of a DEA model influences which decision-making units serve as benchmarks and how inefficiency is assessed relative to the frontier. Consequently, model selection should be explicit and guided by the analytical purpose and empirical context. The Range Directional Model is suitable when directional targets linked to policy or strategy are relevant and translation invariance is required. The Modified Slacks-Based Measure is appropriate when detailed, variable-specific diagnostic information is needed. The Semi-Oriented Radial Measure allows analysts to retain conventional input or output orientations while accommodating non-positive data. Two-Stage DEA models are particularly useful when internal process performance is of interest, albeit at the cost of a stricter efficiency definition.

These conclusions should be interpreted in light of several limitations. The analysis relied on a synthetic dataset, and the expert interviews, while informative, were necessarily limited in number. As a result, the external validity of the findings should be further examined using real-world datasets from different sectors and institutional settings. Nevertheless, the results provide clear lessons for applied efficiency analysis: efficiency outcomes are sensitive to modeling assumptions, and different DEA extensions encode different priorities.

From a broader perspective, this study contributes to the DEA literature by clarifying how alternative treatments of non-positive data translate into differences in efficiency classification and benchmarking outcomes. The findings demonstrate that efficiency assessment is not independent of methodological design choices, and that transparency in model selection is essential, particularly in benchmarking, regulatory, or policy-oriented applications. Future research should build on these results by conducting robustness analyses across datasets, systematically varying model parameters such as directions, orientations, stage weights, and returns to scale, and by integrating sensitivity analysis as a standard component of DEA studies involving non-positive data.

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