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MINIMIZING PERSONNEL COSTS USING LINEAR PROGRAMMING TOOLS ON THE EXAMPLE OF A SERVICE BUSINESS

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Purpose: The aim of this article is to apply theories concerning the optimization of employment levels using linear programming methods based on the assignment problem.
Design/methodology/approach: The research was based on a review of literature on business management and the problem of assignment. In the author's opinion, studies showing the practical possibilities of this type of algorithm fill a cognitive and application gap in the field of management sciences. The research part of the thesis formulates a linear programming model for service activities, concerning the assignment of employees to projects, taking into account their specializations.
Findings: The article expresses the view that the optimal allocation of labor resources has a beneficial impact not only on employment costs but also on strengthening the flexibility of enterprises. Currently, companies must strive for a well-thought-out and rapid allocation of their resources in order to be able to effectively take advantage of market opportunities. The assignment problem can partially support these processes.
The article develops a linear programming model for the allocation of employees to projects, taking into account their specializations. As a result, a combination of allocations was found that generates the lowest employment costs, taking into account other constraints, i.e., employee working time and task duration, employee availability, and required completion dates.
Research limitations/implications: Developing optimization models requires understanding and formulating mathematical functions. For many decision-makers, this is a demotivating factor in exploring the implementation of processes with the support of available programming tools.
Practical implications: The model can be adapted to the conditions of the decision-making task. It can serve as a starting point for various forms of activity in which the individual characteristics of objects and their preferences for the tasks to be performed are known.
Originality/value: It should be emphasized that this article can be a valuable source of knowledge on optimal resource allocation. The proposed algorithm in its presented form can be used in many areas of management. It can also be modified depending on changing input parameters.
Keywords: assignment problem, resource allocation, personnel cost minimization, linear programming.
Category of the paper: Research paper.

1. Introduction

The dynamics of globalization processes, linked to the disappearance of market entry barriers and intensified by the development of advanced technologies, cause turbulence in the functioning of economic entities. The still relevant response to these processes is the flexibility of the enterprise, which allows for effective management in conditions of uncertainty (Eapen, 2010). Companies participating in intense market competition are looking for new ways to redefine the sources of competitive advantage. The proper allocation of labor resources within a company is an important element in achieving organizational flexibility, and when supported by the optimization of these resources aimed at minimizing employment costs, it constitutes an important link in building the company's competitive position.

The aim of this article is to apply theories concerning the optimization of employment levels using linear programming methods, in particular the issue of allocation. The article uses the allocation problem, which in the classical sense boils down to assigning a specific number of objects to tasks (projects) (Krawczyk, 2001). The criterion of optimality can be maximization or minimization of the objective function, depending on the measure of efficiency (time, number of elements, costs, profits, etc.). The article describes the use of the assignment problem in optimizing employment levels and thus reducing personnel costs.

Employment levels, understood as the number of employees, are an important component of total costs in both manufacturing and service companies. The simplest way to reduce them is to decrease the number of employees.

The simplest way to reduce these costs is to reduce the size of the workforce, which leads to a reduction in wages, overheads, and other related costs.

Current and future human resources are a key aspect of management decision-making and the ability to achieve the company's strategic goals. In this context, it is necessary to actively shape and optimally use them (Bień, 2013).

Contemporary organizations face the problem of adapting to constantly changing market conditions. It is necessary to build an organization that uses its resources to respond quickly to changing conditions. This is only possible if the organization has a high degree of flexibility. The ability of a company to adapt to external conditions or internal needs can make the organization more resilient to shocks caused by the dynamics of technological progress, the blurring of boundaries between companies and industries, the reduction of formal barriers to market entry, or the separation of financial flows from the production of material goods and services (Moroz, 2011, following Obłój, 2002). In the area of employment, flexibility usually takes the following form (Beardwell et al., 2004):

- quantitative flexibility – refers to adjusting the level of labor input to the requirements related to the scale of production or the scope of the project,
- task flexibility – involves investing in existing employment resources to develop the ability to perform specific tasks, e.g. through employee training,
- outsourcing – involves moving employment outside the company and contracting external labor resources to perform specific tasks or services,
- wage flexibility – the desired employment structure is achieved through an appropriate salary system.

Optimal use of the potential inherent in human resources means shaping them in a way that allows for the multiplication of the company's value. Rational planning of human resource requirements becomes essential in the process of estimating personnel costs and their derivatives, by avoiding the phenomenon of over-employment. One of the key factors in building a flexible organization capable of continuously responding to the changing needs of the environment and the market seems to be, among other things, efficiency in the process of planning human resources, including the size and structure of employment. In the literature on the subject, employment flexibility refers in particular to the rapid adjustment of the number and profile of employees, who constitute the resources of a given enterprise, to changing conditions and needs.

The article focuses primarily on demonstrating the possibilities of rational management of human resources in terms of assigning personnel to tasks, using linear programming tools. Building and solving linear models allows for the streamlining of the employee selection process based on a defined goal (which varies for different decision-making processes). The developed tool is always available and significantly reduces the time-consuming nature of planning processes. In addition, based on mathematical functions, it helps to avoid mistakes in developing the task assignment structure, which could result in an unnecessary increase in project employment costs. It also allows for quick consideration of changing external parameters, such as hourly rates.

2. Literature rewiev

In the literature, the algorithm for the assignment problem was described in the 1950s in the works of H. Kuhn (1955) and J. Munkers (1957). Since then, the problem has gained popularity and has become the subject of consideration by many scientists. The Web of Science database (WoS) contains over 42,546 scientific articles on this topic, the vast majority of which (over 91%) were written after 2000. Over the last twenty years, an average of 2500 articles on this topic have been added to the database each year. Between 1955 and 2025, the issue of allocation was most frequently described in the categories of Operations Research Management

Science (16.81% of all articles), Engineering Electrical Electronic (16.33%), and Telecommunication (11.03%). The number of articles on the subject of allocation categorized in the 20 most numerous WoS fields is presented in Figure 1.

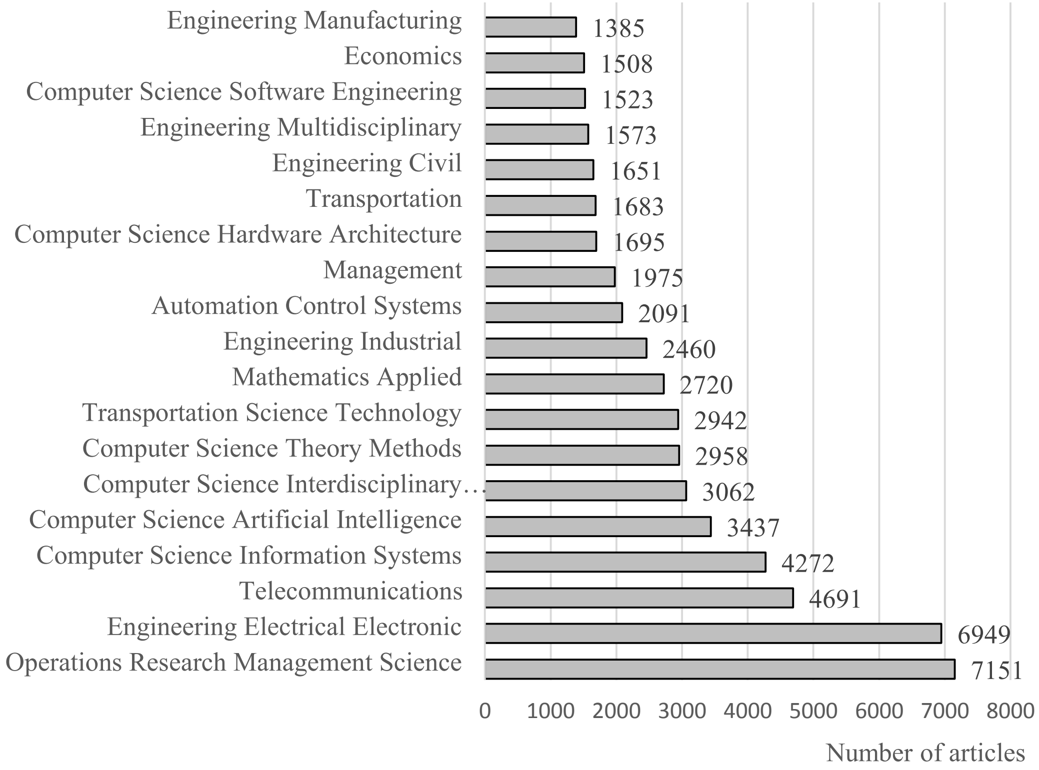

Figure 1. Number of articles published in the WoS database concerning the assignment problem by category (20 most popular).

Source: own study based on data from the WoS database.

Among the numerous applications of the assignment algorithm, there are also those in the fields of management and economics. It is considered as a support in logistics processes, decision-making, resource allocation, and personnel planning. It can apply to decisions made within any company. This is because it refers to improving the process of assigning objects to tasks.

In the literature on the subject, one can find publications referring to many different types of assignment algorithms, such as: the bottleneck AP (Ravindran, Ramaswami, 1977), the generalized AP (Cattrysse, Van Wassenhove, 1992), the quadratic AP (Lawler, 1963), the semi-AP (Kennington, Wang, 1992) and many others. There are also those that are a combination of two or more basic versions (Geetha, Nair, 1993).

Manually assigning employees to tasks without the support of an IT system is a time-consuming task and can generate additional costs related to, for example, delays in starting tasks. Sometimes, manual assignment of tasks to employees, due to its predictability, may give rise to the risk of negative phenomena, such as the risk of corruption (Klimek, Łebkowski, 2012). Assignment issues can be resolved using various methods, including:

The Hungarian algorithm, also known as the Kuhn-Munkres algorithm, is a method for solving assignment problems by finding a matching using several algorithm steps involving the modification of the cost matrix (cf. Kuhn, 1955; Munkers, 1957). The algorithm runs in polynomial time, making it an effective method for many practical applications, including resource management, transportation, production planning, and others;

– linear programming, by formulating the assignment problem as a linear programming problem and solving it using available tools, e.g., the Simplex method (cf. Pamuła, Król, 2013, p. 88; Jędrzejczyk et al., 1996). It is then referred to as a linear assignment problem (LAP).

For advanced decision-making problems, algorithms are used that constitute an individual set of mathematical tools created for the needs of a specific problem. Their complexity and form depend on the constraints and parameters that constitute the input data for a given decision-making problem. Depending on the problem adopted, a multitude of modifications of assignment problems can be observed in the literature (see, among others, Konarzewska, 2020; Duvignau et al., 2026; Cattrysse, Van Wassenhove, 1992; Loiola et al., 2007; Patel, Doshi, 2019). It should be acknowledged that the extensive range of solutions based on the allocation algorithm has several common limitations. These result from the main objection raised against generalized assignment algorithms (GAP), which is the NP-hard nature of the problem. This algorithm poses a fundamental challenge in optimization applied to logistics, resource allocation, and planning. Despite its practical importance, effective GAP solving faces significant limitations due to its inherent computational complexity and the lack of standardized evaluation tools. Furthermore, the time required to find an exact solution grows exponentially with the size of the problem. Exact methods become computationally infeasible for problems of high complexity. Furthermore, it must be acknowledged that many algorithms encounter problems when extended to a larger number of tasks and agents. Implementing an algorithm in two similar decision problems, differing only in the size of the decision variables, may prove impossible due to tool limitations. This may force decision-makers to use increasingly advanced optimization tools, thus generating additional costs for solving such problems.

Several groups of decision-making problems can be identified, the subject of which is the allocation of broadly understood resources. The article uses the problem of allocating labor resources (employees), which can be broadly formulated as follows: m employees must be assigned to complete n orders so that the total allocation costs are as low as possible. The matrix of employee labor costs $C = [c_{ij}]$ is known. In this case, the elements of matrix C represent the cost of the *i*-th employee in completing the *j*-th order. The solution (assignment) matrix $X = [x_{ij}]$ will consist of mxn elements specifying the assignment of a given employee to an order.

Decision variables take a binary form, so each element of the matrix $X = [x_{ij}]$ must belong to the set {0, 1}, i.e. $x_{ij} \in \{0,1\}$, where $x_{ij} = 1$ when the i-th employee is assigned the j-th task and $x_{ij} = 0$ in other cases. Assuming that a given employee can only be assigned to one order (task) and each task can only be performed by one employee, the mathematical notation of the model will look as follows (formulas 1-4) (Pentico, 2007):

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}c_{ij} \rightarrow min \quad (1)$$

$$\sum_{i=1}^{m} x_{ij} = 1 \quad (2)$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad (3)$$

$$x_{ij} \in \{0,1\} \quad (4)$$

The specific form of the model depends on the nature of the parameters provided, resulting from the constraints that must be taken into account in the decision-making process.

3. Methods and results

3.1. Input data

Below is an example of how the work assignment algorithm is used in a company providing small-scale gardening services in the Wielkopolska region.

The decision-making problem consists in assigning 10 employees (i = 1, ..., 10) to complete 14 orders (j = 1, 2, 3, 4) scheduled for 10 consecutive days. The company has a base of 10 employees at its disposal, whose services it uses by hiring them on a contract basis. Each of them has specific key competencies (specializations). Employees should be assigned to orders in such a way that the total cost of employment is as low as possible. Data on the hourly rate (in PLN/h), hourly workload limit (in h), and key competencies of employees are presented in Table 1.

The key competencies required for the specific profile of services provided by the company include: work related to the felling of trees and larger shrubs (chainsaw), work related to cleaning paving stones, facades, etc., requiring the use of a specialized pressure washer (pressure washer), work related to the installation of electrical systems (electricity) and work related to the installation of irrigation systems (irrigation). Other work included in the scope of services provided can be performed interchangeably by employees. In Table 1, the given qualifications are assigned a value of 1 (if the employee has the given skill) or 0 (if the employee does not have the skill).

Table 1.
Characteristics of workers

Worker	Rate (r_i) [PLN/h]	Max_load (T_i) [h]	Core competencies			
			Chainsaw (c_i)	Pressure washer (p_i)	Electricity (e_i)	Irrigation (i_i)
W1	49	80	1	1	0	0
W2	55	60	0	0	1	1
W3	50	55	1	0	1	0
W4	56	70	1	0	0	1
W5	49	20	0	1	0	0
W6	53	30	1	1	1	0
W7	56	50	0	0	1	1
W8	55	80	0	0	1	1
W9	51	40	1	1	0	0
W10	56	23	1	1	1	1

Source: own elaboration.

Table 2 shows the details of individual orders accepted for the next 10 days. They include the order number, place of execution, deadline for completion, and the number of hours needed to complete the order, estimated on the basis of the accepted scope of work. Some orders are delivered on the same day.

Table 2.
Orders details

Order	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12	O13	O14
Place	Poznań	Kórnik	Komorniki	Swarzędz	Mosina	Luboń	Luboń	Komorniki	Poznań	Mosina	Leszno	Mosina	Kostrzyn	Środa Wlkp.
Day	1	2	2	3	3	4	5	6	6	7	7	8	9	10
Time (t_j) [h]	8	12	5	10	10	8	9	10	12	11	10	8	7	9

Source: own elaboration.

A list of required qualifications was specified for each order (Table 3). The required qualifications were assigned a value of 1 (if required) or 0 (if not required).

Table 3.
Qualifications required for the order

Order	Chainsaw (c_j)	Pressure washer (p_j)	Electricity (e_j)	Irrigation (i_j)
O1	0	0	1	0
O2	1	0	0	1
O3	0	1	0	0
O4	0	1	1	0
O5	0	0	1	1
O6	1	0	1	0
O7	0	1	0	1
O8	1	1	1	0
O9	0	0	1	1
O10	0	0	1	1
O11	0	1	1	0
O12	1	1	1	1
O13	0	1	0	1
O14	0	0	1	1

Source: own elaboration.

In addition, the availability declared by each employee for specific order dates is known. Below are the orders in which the employee cannot be involved:

- employee W1 – unavailable for orders O1-O5,
- employee W2 – unavailable for orders O7, O12-O14,
- employee W4 – unavailable for orders O1, O2,
- employee W5 – unavailable for orders O4, O5,
- employee W6 – unavailable for orders O2, O3, O12,
- employee W7 – unavailable for orders O1,
- employee W8 – unavailable for order O6,
- employee W9 – unavailable for order O11,
- employee W10 – unavailable for order O1, O5.

In addition, it is known that each order must be carried out by at least two employees.

3.2. Solution of the model

The decision variables are presented in the form of a matrix $X = [x_{ij}]$ (Table 4). The variables take the form x_{ij}, where i denotes the employee number and j denotes the order number. In accordance with the assumptions of the assignment problem, they take a binary form, i.e., 1 if the i-th employee is involved in the j-th order and 0 if there is no such involvement.

Table 4.
Symbols of decision variables

	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12	O13	O14
W1	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	$x_{1,10}$	$x_{1,11}$	$x_{1,12}$	$x_{1,13}$	$x_{1,14}$
W2	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	$x_{2,10}$	$x_{2,11}$	$x_{2,12}$	$x_{2,13}$	$x_{2,14}$
W3	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}	$x_{3,10}$	$x_{3,11}$	$x_{3,12}$	$x_{3,13}$	$x_{3,14}$
W4	x_{41}	x_{42}	x_{43}	x_{44}	x_{45}	x_{46}	x_{47}	x_{48}	x_{49}	$x_{4,10}$	$x_{4,11}$	$x_{4,12}$	$x_{4,13}$	$x_{4,14}$
W5	x_{51}	x_{52}	x_{53}	x_{54}	x_{55}	x_{56}	x_{57}	x_{58}	x_{59}	$x_{5,10}$	$x_{5,11}$	$x_{5,12}$	$x_{5,13}$	$x_{5,14}$
W6	x_{61}	x_{62}	x_{63}	x_{64}	x_{65}	x_{66}	x_{67}	x_{68}	x_{69}	$x_{6,10}$	$x_{6,11}$	$x_{6,12}$	$x_{6,13}$	$x_{6,14}$
W7	x_{71}	x_{72}	x_{73}	x_{74}	x_{75}	x_{76}	x_{77}	x_{78}	x_{79}	$x_{7,10}$	$x_{7,11}$	$x_{7,12}$	$x_{7,13}$	$x_{7,14}$
W8	x_{81}	x_{82}	x_{83}	x_{84}	x_{85}	x_{86}	x_{87}	x_{88}	x_{89}	$x_{8,10}$	$x_{8,11}$	$x_{8,12}$	$x_{8,13}$	$x_{8,14}$
W9	x_{91}	x_{92}	x_{93}	x_{94}	x_{95}	x_{96}	x_{97}	x_{98}	x_{99}	$x_{9,10}$	$x_{9,11}$	$x_{9,12}$	$x_{9,13}$	$x_{9,14}$
W10	$x_{10,1}$	$x_{10,2}$	$x_{10,3}$	$x_{10,4}$	$x_{10,5}$	$x_{10,6}$	$x_{10,7}$	$x_{10,8}$	$x_{10,9}$	$x_{10,10}$	$x_{10,11}$	$x_{10,12}$	$x_{10,13}$	$x_{10,14}$

Source: own elaboration.

The objective of the decision problem presented will be to minimize the function representing the sum of the products of the hourly rate of employees (r_i), the duration of individual orders (t_j), and the matrix of decision variables X. Finally, it will take the form as formula 5:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} r_i t_j x_{ij} \rightarrow min \tag{5}$$

Based on the list of key competencies for a given task (c_j, p_j, e_j, i_j) and the list of key competencies for each employee (c_i, p_i, e_i, i_i), a summary of key competencies for each employee and task (c_{ij}, p_{ij}, e_{ij}, i_{ij}) was developed according to the following notation (see formulas 6-9):

$$c_{ij} = \begin{cases} 0, c_i \cdot c_j = 0 \\ 1, c_i \cdot c_j = 1 \end{cases} \quad (6)$$

$$p_{ij} = \begin{cases} 0, p_i \cdot p_j = 0 \\ 1, p_i \cdot p_j = 1 \end{cases} \quad (7)$$

$$e_{ij} = \begin{cases} 0, e_i \cdot e_j = 0 \\ 1, e_i \cdot e_j = 1 \end{cases} \quad (8)$$

$$i_{ij} = \begin{cases} 0, i_i \cdot i_j = 0 \\ 1, i_i \cdot i_j = 1 \end{cases} \quad (9)$$

The results are presented in Table 5, but due to the complex structure of the data, only the results for key competencies for the execution of a given order have been included, i.e. those that meet the following conditions: $c_j = 1$, $p_j = 1$, $e_j = 1$, $i_j = 1$. This means that if an employee has the required qualification for a given order, the assigned value is 1. They are therefore suitable for the work assignment for a given order and can be taken into account in the assignment. If an employee does not have the required qualification for a given order, then a value of 0 is entered in the appropriate place in the table. This means that the value assigned to the employee's key competence in Table 1 does not match the value assigned to the key competence for the order (Table 2).

The values shown in Table 5 were obtained using the "IF" formula in a spreadsheet. For each competency, an assessment was made of the degree of alignment between the employee's skills and the requirements of the job. If both answers were affirmative, the value 1 was entered, in other cases the value 0 was entered, e.g. $c_{16} = 1$, because the value c_i for i = 1 is 1 (Table 1) and the value c_j for j = 6 is 1 (Table 3); $p_{28} = 0$, because the value of c_i for i = 2 is 0 (Table 1), and the value of c_j for j = 8 is 1 (Table 3); $e_{2,12} = 1$, because the value of c_i for i = 2 is 1 (Table 1), and the value of c_j for j = 12 is 1 (Table 3), etc. The "IF" formula in the spreadsheet was used to calculate the values in Table 5. For the purposes of further calculations, the existence of matrix K is assumed, to which belong those elements c_{ij}, p_{ij}, e_{ij} and i_{ij} for which $c_j = 1$, $p_j = 1$, ej = 1, $i_j = 1$.

Table 5.
The key competence for the order (c_{ij}, p_{ij}, e_{ij}, i_{ij})

	O1	O2		O3	O4		O5		O6		O7		O8			O9		O10		O11		O12				O13		O14	
	electricity (e_{i1})	chainsaw (c_{i2})	irrigation (i_{i2})	pressure washer(p_{i3})	pressure washer (p_{i4})	electricity (e_{i4})	electricity (e_{i5})	irrigation (i_{i2})	chainsaw (c_{i6})	electricity (e_{i6})	pressure washer(p_{i7})	irrigation (i_{i7})	chainsaw (c_{i8})	pressure washer (p_{i8})	electricity (e_{i8})	electricity (c_{i9})	irrigation (i_{i9})	electricity ($e_{i,10}$)	irrigation ($i_{i,10}$)	pressure washer ($p_{i,11}$)	electricity ($e_{i,11}$)	chainsaw ($c_{i,12}$)	pressure washer ($p_{i,12}$)	electricity ($e_{i,12}$)	irrigation ($i_{i,12}$)	pressure washer ($p_{i,13}$)	irrigation ($i_{i,13}$)	electricity ($e_{i,14}$)	irrigation ($i_{i,14}$)
W1	0	1	0	1	1	0	0	0	1	0	1	0	1	1	0	0	0	0	0	1	0	1	1	0	0	1	0	0	0
W2	1	0	1	0	0	1	1	1	0	1	0	1	0	0	1	1	1	1	1	0	1	0	0	1	1	0	1	1	1
W3	1	1	0	0	0	1	1	0	1	1	0	0	1	0	1	1	0	1	0	0	1	1	0	1	0	0	0	1	0
W4	0	1	1	0	0	0	0	1	1	0	0	1	1	0	0	0	1	0	1	0	0	1	0	0	1	0	1	0	1
W5	0	0	0	1	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	0
W6	1	1	0	1	1	1	1	0	1	1	1	0	1	1	1	1	0	1	0	1	1	1	1	1	0	1	0	1	0
W7	1	0	1	0	0	1	1	1	0	1	0	1	0	0	1	1	1	1	1	0	1	0	0	1	1	0	1	1	1
W8	1	0	1	0	0	1	1	1	0	1	0	1	0	0	1	1	1	1	1	0	1	0	0	1	1	0	1	1	1
W9	0	1	0	1	1	0	0	0	1	0	1	0	1	1	0	0	0	0	0	1	0	1	1	0	0	1	0	0	0
W10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Source: own elaboration.

We begin formulating the functions related to the constraints of the decision-making problem by assigning tasks according to the qualifications possessed and required by employees. The data contained in Table 5 (matrix K) was used for this purpose. It should be noted that the final solution will require at least one employee with the competence required for the order. For each key competence, at least one employee who possesses it must be assigned. This group of conditions will take the form shown in formulas 10-13.

$$\bigwedge_{j \in K} \sum_{i=1}^{14} c_{ij} x_{ij} \geq 1 \tag{10}$$

$$\bigwedge_{j \in K} \sum_{i=1}^{14} p_{ij} x_{ij} \geq 1 \tag{11}$$

$$\bigwedge_{j \in K} \sum_{i=1}^{14} e_{ij} x_{ij} \geq 1 \tag{12}$$

$$\bigwedge_{j \in K} \sum_{i=1}^{14} i_{ij} x_{ij} \geq 1 \tag{13}$$

It follows that for each key competence required for the order, listed in Table 5, the sum of the products of its individual columns and the corresponding column of decision variables (for the appropriate order number) will be at least 1. Let us take the first column of Table 5 as an example. The limiting condition here will take the form $\sum_{i=1}^{14} e_{i1} x_{i1} \geq 1$, more precisely

$$0x_{11} + +1x_{21} + 1x_{31} + 0x_{41} + 0x_{51} + 1x_{61} 1x_{71} + 1x_{81} + 0x_{91} + 1x_{10,1} \geq 1$$

Using the appropriate spreadsheet formula, we proceed in the same way for the remaining columns of Table 5.

Taking into account other constraints resulting from the availability of employees on specific dates, regarding the maximum permissible workload of employees (formulas 14-17), the number of employees to be assigned (formula 18), overlapping order dates (formula 19), employee availability for orders (formula 20), and the binary nature of the variables in matrix X (formula 21), the rest of the problem will take the following mathematical form:

$$x_{i2} + x_{i3} \leq 1 \tag{14}$$

$$x_{i4} + x_{i5} \leq 1 \tag{15}$$

$$x_{i8} + x_{i9} \leq 1 \tag{16}$$

$$x_{i,10} + x_{i,11} \leq 1 \tag{17}$$

$$\sum_{i=1}^{14} x_{ij} \geq 2 \tag{18}$$

$$\sum_{j=1}^{10} x_{ij} t_j \leq T_i \tag{19}$$

$$\begin{gathered} x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{27} + x_{2,12} + x_{2,13} + x_{2,14} + x_{41} + \\ + x_{42} + x_{54} + x_{55} + x_{62} + x_{63} + x_{6,12} + x_{71} + x_{86} + x_{9,11} + x_{10,1} + x_{10,5} = 0 \end{gathered} \tag{20}$$

$$x_{ij} \in \{0,1\} \tag{21}$$

The formulated linear programming task was solved using the Solver module included in MS Excel, and the results are presented in Table 6.

Table 6.
Optimal allocation of employees to orders based on the criterion of minimum personnel costs

	O1	O2	O3	O4	O5	O6	O7	O8	O9	O10	O11	O12	O13	O14	Total cost
W1	0	0	0	0	0	1	1	0	1	0	1	1	1	1	**3087**
W2	0	0	0	1	0	1	0	1	0	0	1	0	0	0	**2090**
W3	1	0	1	0	1	0	1	0	1	0	0	0	0	1	**2650**
W4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
W5	0	0	0	0	0	0	0	1	0	0	0	0	1	0	**833**
W6	1	0	0	0	1	0	0	0	0	0	0	0	0	0	**954**
W7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	**0**
W8	0	1	0	0	0	0	0	0	0	1	0	1	0	0	**1705**
W9	0	1	0	1	0	0	0	0	0	1	0	0	0	0	**1683**
W10	0	0	1	0	0	0	0	0	0	0	0	0	0	0	**280**
Total cost	**824**	**1272**	**530**	**1060**	**1030**	**832**	**891**	**1040**	**1188**	**1166**	**1040**	**832**	**686**	**891**	**13282**

Source: own elaboration.

According to the results obtained, the optimal plan for assigning employees to orders is as follows:

– employee W1 has been assigned to orders O6, O7, O9, O11, O12, O13, O14,
– employee W2 has been assigned to orders O4, O6, O8, O11,
– employee W3 has been assigned to orders O1, O2, O5, O7, O9, O14,
– employee W5 has been assigned to ordersO8, O13,
– employee W6 has been assigned to orders O1, O5,
– employee W8 has been assigned to orders O2, O10, O12,
– employee W9 has been assigned to orders O2, O4, O10,
– employee W10 has been assigned to order O3.

All optimization constraints have been met, and the total labor costs amount to PLN 13,282 (Table 6). Table 6 also shows the salary costs for individual employees and for orders. The data in Table 6 shows that exactly two employees were involved in each order. In addition, two employees (W4 and W7) remain unassigned to orders. ($\sum_{j}^{m} x_{4j} = 0; \sum_{j=1}^{m} x_{7j} = 0$).

It is worth noting that the task allows for further modifications, e.g. in terms of the number of employees assigned to tasks, so that, for example, each employee is involved in at least one assignment. In the example discussed, this was not a necessary condition, as the employer selects contractors from a specific pool of employees, hiring them on a contract basis. It should also be remembered that additional limits will generate higher employment costs. The choice of approach and limits depends here on the individual preferences of the decision-maker.

4. Conclusions

To sum up the considerations presented in this article, it should be acknowledged that through the proper allocation of resources within a company, it is possible to achieve the lowest employment costs without having to search through a matrix of all possible assignment combinations. In most companies, the process of assigning employees to tasks is done intuitively (manually), which does not guarantee the greatest possible financial benefits. With the support of appropriate IT tools, the process of obtaining the right assignment is significantly shortened. The linear programming model constructed and solved in the article can have a wide range of applications. Although the article describes its operation on the example of a service activity (gardening), it can be successfully applied in many other types of activities.All that is required is for the decision-making task to be formulated as a search for such an allocation of objects to tasks that generates the lowest cost and has the following constraints:

- maximum workload of objects (employees) and required for tasks (projects),
- specialization (competence) possessed by objects and required for tasks,
- task completion deadlines and object availability.

Additionally, it is worth emphasizing that the presented model is characterized by a high degree of flexibility and can be adapted to the conditions of the decision-making task. It can serve as a starting point for various forms of activity in which the individual characteristics of objects and their preferences for the tasks to be performed are known. Examples of applications of the presented model include problems of staffing customs posts with customs officers, taking into account the hierarchy of work (rank), the assignment of medical personnel to procedures in accordance with their qualifications, and the assignment of judges to adjudicate cases in accordance with their preferences or experience.

In line with the above, the article proposes the use of linear programming aimed at finding the lowest employment costs. The proposed algorithm focuses on the use of key employee competencies. The use of these unique abilities is an important link in the management of company resources, especially human resources. Identifying and purposefully placing employees with outstanding competencies in strategic areas (e.g., key development projects, innovation teams) allows for maximum efficiency. These employees are often the driving force behind innovation, contributing to shorter completion times for complex tasks and improving the quality of products or services offered. These competencies may include highly developed skills, knowledge, or attitudes that distinguish them from accepted market or internal company standards. These include, for example, in-depth technical knowledge, soft skills, or specific cognitive or adaptive abilities. Depending on the type of decision-making problem, the algorithm may apply to all of the above-mentioned skills. It can support the search for effective ways to use the company's resources, which can lead to competitive advantage and faster response to market opportunities. It can therefore be an important link in building organizational flexibility.

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