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INTERRELATIONSHIPS BETWEEN SELECTED INNOVATION DIFFUSION MODELS USING SYSTEM DYNAMICS

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Purpose: The aim of this publication is to identify the distinguishing features of innovation diffusion models. This objective will be achieved based on five innovation diffusion models found in the literature: the logistic curve, the Bass model, the source model, the contact model, and the contact-source model.
Methodology: System dynamics modeling using AnyLogic and AnyLogic Cloud will be applied for this purpose. To describe and compare the models, a series of simulations was carried out, which allowed us to answer six research questions concerning, among other things, changes in model characteristics under different coefficient values, the population of potential users, and the initial number of innovative technology adopters.
Findings: The outcome of this work is a more accurate understanding of diffusion models, which are usually presented as mathematical equations. This may serve as a useful tool for those involved in marketing and development activities in the process of creating organizational strategies.
Originality/value: This study introduces a novel application of system dynamics to compare and mathematically characterize five innovation diffusion models, focusing on how parameter variations influence their behavior. The approach offers fresh insights into model similarities, differences, and applicability, providing a stronger foundation for their practical use in research and management.
Keywords: simulation modeling, system dynamics, innovation diffusion models, mathematical descriptions of innovation diffusion.
Category of the paper: Research paper.

1. Introduction

Innovation plays a key role in the development and competitiveness of firms in a changing market environment. It is defined as the introduction of novelties or changes and encompasses not only advanced technologies but also various industries (Aslam et al., 2023; Szafranowicz, 2019). Three interrelated concepts can be distinguished: invention (absolute novelty), innovation (implementation of the invention), and imitation (dissemination of innovation) (Wasilewska, 2012). Contemporary approaches to innovation, such as open innovation, enable firms to draw from external sources of knowledge, thereby increasing their competitiveness. Innovation affects companies by enhancing their ability to adapt and survive in dynamic markets (Niedzielski, Henhappel, 2015). The introduction of innovation may also improve market position through collaboration with other entities (Karaś et al., 2021).

Innovation diffusion refers to the process by which new ideas, technologies, or practices spread within and across organizations over time, typically through communication channels among members of a social system (Dearing, Cox, 2018; Rogers, 1983). In organizational contexts, successful diffusion can lead to improved performance, competitive advantage, and greater adaptive capacity as innovations are adopted and integrated into operations (Hassan et al., 2024; Steiber et al., 2021). Diffusion is not always straightforward—it may be hindered by technological, social, and learning barriers as well as institutional and contextual factors such as infrastructure or organizational culture (MacVaugh, Schiavone, 2010; Vargo et al., 2020).

Innovation diffusion is essential for disseminating new technologies, practices, or ideas across societies and organizations, enabling progress and the tackling of complex challenges in many industries and domains. The process is relevant in numerous sectors, including healthcare, industry, mining, and the public sector (Cai et al., 2022; de Vries et al., 2018; Fichter, Clausen, 2021).

The diffusion process can be modeled under different assumptions—deterministic, stochastic, or chaotic—depending on the chosen approach and level of detail. Classical models, such as the Bass model, portray diffusion deterministically using differential equations to forecast the number of adoptions over time, assuming predictability and the absence of randomness in consumer behavior (Cai et al., 2022). In reality, diffusion is typically shaped by diverse random external and internal factors, including changes in preferences, competitive actions, and unforeseen market events. As a result, stochastic models are used to better capture variability and uncertainty in these processes (Singhal et al., 2019). In more complex analyses, innovation diffusion may exhibit features of deterministic chaos, where small initial changes lead to large differences in outcomes. Such systems can combine elements of randomness with deterministic chaos—sometimes described as “evolutionary chaos” (Silverberg, Lehnert, 1993).

Both mathematical and qualitative descriptions of innovation diffusion models exist in the literature. Classical models are formally expressed through differential and statistical equations, yet their assumptions and interpretations often have a qualitative character, describing social processes, actor roles, communication channels, or implementation barriers (Guidolin, Manfredi, 2023; Mahajan, Peterson, 1985). Contemporary approaches increasingly emphasize the importance of social context, service ecosystems, and institutions. This provides theoretical and qualitative frameworks that go beyond single actors or simple flows of innovation and focus on complex interactions and institutional change (Vargo et al., 2020). In practice, qualitative studies are leveraged to analyze the target market for an innovation, identify key factors affecting diffusion, and select appropriate mathematical models for subsequent quantitative analysis (Frenzel, Grupp, 2009).

Deterministic mathematical models—especially those based on differential equations—are widely used to describe innovation diffusion and form the foundation of many analyses in this field. They make it possible to capture general trends associated with the spread of innovation in society, often assuming population homogeneity and process predictability. In practice, deterministic models are applied to forecast adoption of new products, plan marketing strategies, and assess the effectiveness of undertaken actions (Guidolin, Manfredi, 2023; Karmeshu, Jain, 1997; Mahajan, Peterson, 1985).

The relationships presented by Bass, Rogers, and Moore, as well as earlier models such as the logistic curve, remain relevant and are frequently used. They are applied to forecasting and describing the pace of diffusion of selected technologies, new organizational or marketing methods within enterprises and in consumer markets, across sectors such as logistics, commerce, and manufacturing (Fant et al., 2023; Fuhua, 2009; Kang, 2021; Klincewicz, 2011; Pratiwi et al., 2022; Yang, Williams, 2008).

Despite the broad use of these models and the possibility of developing new ones tailored to specific diffusion contexts, it is difficult to find compilations that present their interrelationships and differences. Proper comparison and identification of the areas where individual models are most useful facilitate choosing the model that most accurately reflects reality. This should be crucial for those tasked with forecasting product diffusion in the market—and thus revenues from sales.

Innovation diffusion can therefore be described in a deterministic manner, very often using a mathematical formulation. The literature includes a range of mathematical models that depict this process and enable forecasting of the future outcomes of actions undertaken by an enterprise.

In this publication, we attempt to detect and define the relationships between five selected mathematical models describing innovation diffusion: the logistic curve, the Bass model, the source model, the contact model, and the contact–source model (Bradley, 1999; Fant et al., 2023; Kot et al., 1993). This will help to identify differences as well as the strengths and weaknesses of each model and to justify their selection for particular product groups.

The objective will be achieved using simulation modeling in the system dynamics convention with the AnyLogic software.

2. Methods

The five innovation diffusion models outlined in the previous section must be carefully analyzed in order to specify the differences and similarities in their functioning. The appropriate approach for analyzing these models is one consistent with their structure. Since the structure of innovation diffusion models is expressed in the form of differential equations, the method employed here is fundamentally based on the analysis of differential equation systems—namely, the System Dynamics (SD) modeling method. This choice follows from the fact that innovation diffusion concerns social systems, and the analysis is performed at a high level of generality over long periods of time. In the scientific domain, the accepted method for analyzing such systems—long-term, strategic-level, applied to large populations, and described in terms of differential relationships—is system dynamics (Borshchev, Filippov, 2004; Grigoryev, 2021; Größler et al., 2008; Lidberg et al., 2020).

System dynamics modeling combines both qualitative and quantitative approaches. To represent a system as faithfully as possible through a model, it is first necessary to identify, by means of causal loop diagrams (CLDs), the interdependencies among the factors influencing the system, and then transform them into a quantitative stock-and-flow diagram (SFD) (Harmoza, 2006; Ong et al., 2022; Poles, 2013; Walters et al., 2016; Wang et al., 2024).

Causal loop diagrams (CLDs) illustrate only the mutual influences among the system's elements. They indicate the direction of influence and whether it is reinforcing or diminishing. Furthermore, when constructing such diagrams, feedback loops emerge that determine whether a given closed cycle of relationships is reinforcing, balancing, or stabilizing (Gładysz, Santarek, 2017; Größler et al., 2008).

The purpose of stock-and-flow diagrams (SFDs) is to represent the system as a set of relations between stock and flow objects. Stocks reflect the size of a given population (e.g., inventory levels, waste, raw materials in production), while flows represent the transitions between them. Flows can either remain constant over time or vary depending on the current parameters of the system (Grobel-Kijanka, 2016; Poles, 2013).

The approach adopted in this study is grounded in the well-established theory of innovation diffusion, originally developed by Rogers, in which the mechanisms of social influence and adopter categories (innovators, early adopters, early majority, late majority, laggards) play a central role (Rogers, 2003). The literature emphasizes that the diffusion process is not merely the outcome of individual consumer decisions but depends on feedback loops within the social and market system. Classical models, such as the Bass model, provide a mathematical formalization of these interdependencies and are widely applied to describe the dynamics of adoption of new products and technologies (Bass, 1969).

System Dynamics (SD) offers a framework that enables the interpretation and extension of these concepts by explicitly representing causal structures. The diffusion process can be conceptualized in terms of feedback loops: a reinforcing loop (the role of interpersonal communication and the "word-of-mouth" effect), which drives adoption growth, and a balancing loop, associated with market saturation and the depletion of potential adopters. The SD formalism thus allows not only for replicating the classical adoption curves but also for examining shifts in loop dominance over time, which extends theoretical analysis beyond static models (Sterman, 2000; Forrester, 1961).

System Dynamics constitutes an appropriate and theoretically grounded paradigm for modeling innovation diffusion at a strategic, aggregate level.

This integration is supported by the following arguments, derived from the literature:

Modeling social systems: as noted by Grigoryev (2021) and Größler, Thun, and Milling (2008), SD is particularly useful for modeling complex social systems characterized by nonlinearities, feedback loops, and delays. The innovation diffusion process, as a social phenomenon, fits this description precisely, as it involves interactions among user populations, the influence of communication, and imitation effects.

Long-term and strategic analysis: SD is recommended for long-horizon analyses, where the focus lies on aggregate trends rather than individual-level behavioral details (Borshchev, Filippov, 2004; Lidberg et al., 2020). This is consistent with the aim of classical deterministic diffusion models (e.g., Bass, logistic), which describe macroscopic adoption trends over time rather than microscopic, stochastic decisions.

Representation of causal structures: the foundation of SD is the identification and formalization of feedback loops that govern system behavior (Sterman, 2000; Gladysz, Santarek, 2017; Größler et al., 2008). In the context of diffusion, this corresponds to positive feedback ("imitation" – more users attract additional ones) and negative feedback ("market saturation" – a shrinking pool of potential adopters slows growth). The transformation of diffusion equations into Stock-Flow Diagrams constitutes a direct operationalization of this theory, enabling visualization and simulation of causal structures.

Integration of qualitative and quantitative perspectives: SD provides a framework that combines qualitative system understanding (through Causal Loop Diagrams – CLDs) with quantitative simulation of system behavior (through Stock-Flow Diagrams – SFDs) (Poles, 2013; Walters et al., 2016). This approach aligns with the dual (qualitative–quantitative) character of diffusion research, described in the Introduction (Guidolin, Manfredi, 2023; Mahajan, Peterson, 1985).

Moreover, an additional advantage of SD lies in its capacity to integrate demand- and supply-side processes within a single model. Unlike purely operational approaches, SD makes it possible to capture the interactions between social influences and resource constraints, production capacities, or marketing strategies. The literature emphasizes that such an extension enhances the analytical potential of diffusion models, enabling the study of policies supporting innovation introduction under conditions of complex causal interdependencies (Peres, Muller, Mahajan, 2010). Our study follows this tradition, employing System Dynamics not only as a computational tool but also as a conceptual framework for examining and interpreting diffusion processes.

The objective of this publication is to investigate the interrelationships among the most frequently cited innovation diffusion models in the literature—namely, the logistic curve, the Bass model, the source model, the contact model, and the contact–source model (Fant et al., 2023; Kot et al., 1993; Tao, Deyong, 2012). This will be achieved by transforming the differential equations characterizing the flows in each diffusion model into simulation models using system dynamics. The implementation is carried out in the AnyLogic simulation environment. The figure below (Figure 1) shows the stages of implementation that were carried out during the work.

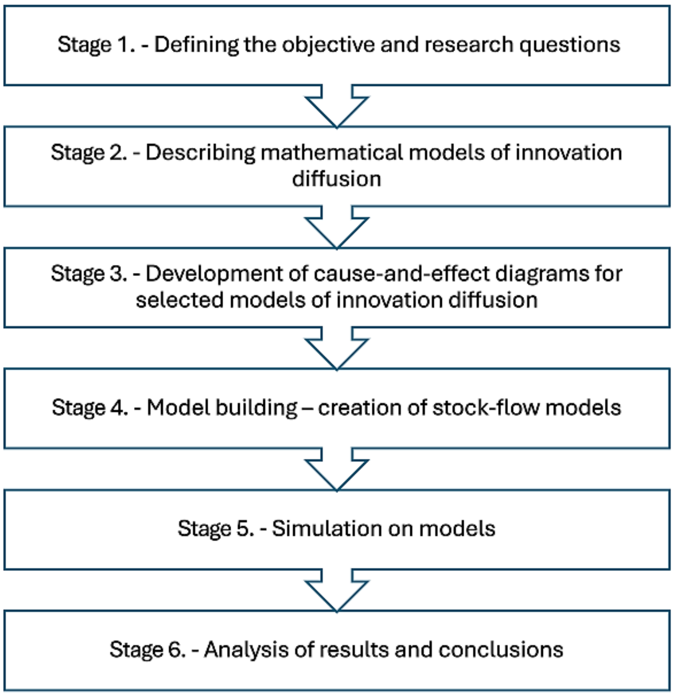

Figure 1. Stages of system dynamics application in building and comparing innovation diffusion models.

Source: authors' own elaboration.

To compare the operating principles of different models and to gain insight into the properties arising from the application of each in practice, the following research questions were posed:

1. How do increases and decreases in the initial parameters of the models affect the total diffusion time?
2. How does the size of the recipient population affect the total diffusion time?
3. How does the initial number of users affect the total diffusion time?

4. Is it possible to determine such initial parameters for the analyzed models that the total diffusion time becomes identical?
5. How do diffusion times vary across Rogers' adopter categories in the averaged models?
6. In which of Rogers' adopter categories does the maximum diffusion rate occur in the averaged models?

Answers to the above research questions will be obtained through the analysis of results from specific simulation scenarios. Each scenario of initial parameter change will be conducted separately for the given models, with the corresponding outcomes recorded. Importantly, in a given scenario only one variable will be tested for its effect on the entire system, while all other conditions remain unchanged (*ceteris paribus*).

It is also important to recognize the limitations inherent in this modeling approach. By applying system dynamics in this way, only models with constant initial parameters—unchanging throughout the simulation—can be analyzed. In practice, population behaviors and the effectiveness of innovation promoters' actions may evolve over time and may be either dependent or independent of the promoter. These dynamics are not accounted for in the constructed models and may therefore affect the final results. Furthermore, the obtained relationships are subject to errors resulting from rounding in the AnyLogic software as well as from the numerical methods used to solve differential equations.

Integration algorithm ("choice of solver") and step size ("discretization step size") – by default, SD models in AnyLogic employ the Euler algorithm with a fixed time step. Variability in the step size or an excessively large step may lead to integration errors and numerical artifacts (e.g., oscillations, instabilities), as emphasized both in the AnyLogic documentation and in SD literature. As Grigoryev (2021) notes in the AnyLogic manual, the choice of integration method (Euler vs. Runge-Kutta) and step size (dt) can significantly affect simulation accuracy, particularly for "stiff" or highly dynamic systems. In our study, all models were run with a fixed time step of dt = 0.001 day. Given the diffusion time scale of several tens of days, this value was deemed sufficiently small to ensure result stability and to minimize discretization error for these specific models.

Numerical stability – simple diffusion models, such as the logistic curve or the Bass model, are generally numerically stable for the parameter ranges considered and when using a sufficiently small step size. Nevertheless, it is worth acknowledging this as a potential limitation, especially in extreme scenarios (e.g., very high values of parameters r, p, q), which could theoretically result in instability. In our simulations, no such behavior was observed, suggesting that the chosen parameter ranges and step size were appropriate.

Rounding error – this limitation is typical of all computer simulations and was noted explicitly. Its impact on the qualitative results presented in this paper (curve shapes, parameter relationships) is negligible, though it may have nonzero significance in highly precise, absolute numerical comparisons across different simulation runs.

AnyLogic provides a range of integration methods. The optimal choice depends on the research objective. For models with slowly varying parameters over time, without sharp peaks in the data, and when predicted trajectories are described by C1-class curves, Euler integration is sufficient and represents a sound compromise between speed and accuracy. It is often applied in systems where large errors arise from aggregation, measurement inaccuracy, simplifications, or incomplete information. Conversely, in physical-system models and simple conceptual models—particularly those involving oscillations or rapid value changes—the Runge-Kutta (R-K) method is frequently preferred.

For most models of social systems, both integration methods (Euler and R-K) tend to produce comparable results.

To summary - the dynamic diffusion models developed for this study were solved using the following methods: differential equations with Euler integration, algebraic equations with Newton's method, and mixed equations with a combination of R-K and Newton's method. The following numerical parameters were adopted: for values close to zero, where relative precision is not feasible, an absolute tolerance of 1E-5 was applied; the relative accuracy guiding the solution of equations with variable step methods was set at 1E-5; the time-step precision for determining switching points during equation solving was set at 1E-5; and a fixed time step of 1E-3 was used for the R-K method.

The results are subject to numerical error, whose magnitude depended on the selected algorithm (Euler) and time step (dt = 0.001 day). Employing more advanced methods (e.g., R-K) could potentially increase accuracy but at the cost of longer computation times. For the purposes of this study—a comparative analysis of model behavior—the adopted configuration was deemed fully sufficient.

3. Results

Between the stage of developing models within the system dynamics framework and conducting specific simulations, it is important to verify the consistency and variability of the models depending on the input data provided.

A useful tool for this purpose is the AnyLogic Cloud computing environment, which enables the creation of experiments. Within the experiment settings, one can specify the ranges and step sizes (iterations) of the input parameters for which simulations will be executed (Grigoryev, 2021). As a result, it is possible to obtain dependency plots displaying all iterations of the simulations, thus allowing the assessment of how input parameters influence the generated outcomes.

Table 1 below presents the differential equations characterizing the flow magnitude per unit of time for the selected innovation diffusion models. Based on these equations, the flows and other elements characteristic of system dynamics were reconstructed.

Table 1.
Descriptions of flows in the analyzed innovation diffusion models

Model name	Equation	Equation number
The Logistic curve	$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$ where: N – population size at time t, t – time, r – growth rate coefficient – describes the combined effect of factors influencing the diffusion of innovation, K – environmental capacity.	(1)
The Bass model	$$\frac{dN(t)}{dt} = p[m - N(t)] + \frac{q}{m}N(t)[m - N(t)]$$ where: N(t) – number of users using the technology at time t, t – time, m – maximum number of technology users, p – innovation coefficient - reflects the probability that consumers will decide to adopt a new product regardless of the influence of other users or under the influence of external factors such as advertising or marketing activities, q – imitation coefficient - measures the influence of existing users on the decisions of potential buyers. Consumers are more likely to make a purchase decision when they see that others have already done so.	(2)
The source model	$$\frac{dy}{dt} = k(n - y)$$ where: y – number of users using the technology at a given time, t – time, n – size of the population under consideration, k – proportionality coefficient - describes the rate or intensity of innovation adoption through the use of marketing and advertising activities.	(3)
The contact model	$$\frac{dy}{dt} = ay(n - y)$$ where: y – number of users using the technology at a given time, t – time, f – effectiveness of persuading supporters of innovation, l – number of contacts of a given person per unit of time, $$a = \frac{fl}{n} = const.$$	(4) (5)
The contact-source model	$$\frac{dy}{dt} = ay(n - y) + k(n - y)$$	(6)

Source: authors' elaboration based on (Fant et al. (2023); Kot et al. (1993); Tao, Deyong (2012)).

The following subsections present the results for the developed models under various criteria.

For example, using the Bass model, the dependencies arising from changes in the parameters of innovation (p) and imitation (q) are illustrated (Figure 2a and Figure 2b). The plot demonstrates that changes in initial parameters significantly affect the shape of both the cumulative distribution function and the probability density function. In practice, this reflects the impact of different activities supporting the diffusion of innovation. Beyond the number of such activities, the financial resources allocated—for instance, to product advertising campaigns in the media—also play a crucial role. This implies that the diffusion of innovation proceeds at different times and with varying intensity in a given unit of time.

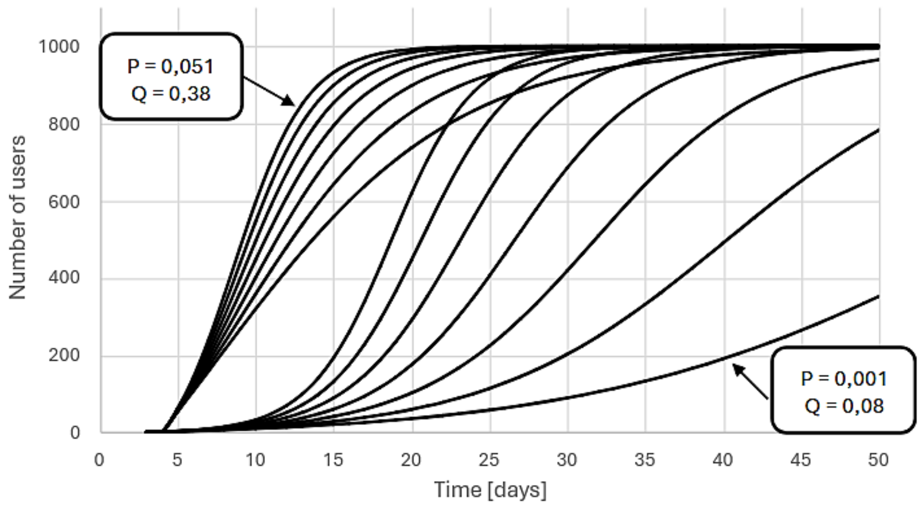

Figure 2a. Cumulative distribution curve of the Bass model under varying input parameters (N = 1000, Np = 1, p ∈ <0.001; 0.1> step 0.05, q ∈ <0.08; 0.4> step 0.05).

Source: authors' elaboration using AnyLogic Cloud.

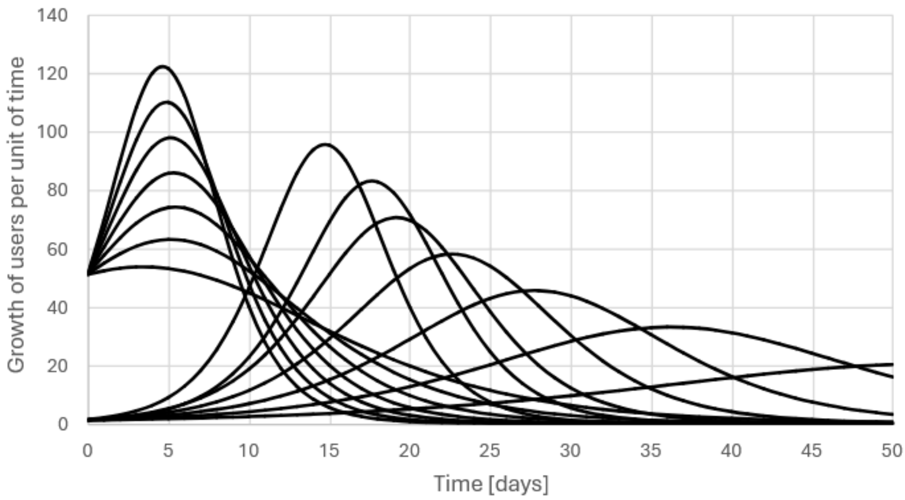

Figure 2b. Probability density curve of the Bass model under varying input parameters (N = 1000, Np = 1, p ∈ <0.001; 0.1> step 0.05, q ∈ <0.08; 0.4> step 0.05).

Source: authors' elaboration using AnyLogic Cloud.

The remaining diffusion models were analyzed in a similar way. Figures below (Figure 3a and 3b) present the cumulative distribution and density curves depending on the changes in initial parameters. In practice, these correspond to changes in the intensity of undertaken actions or in the size of the analyzed population, as specified in Table 1.

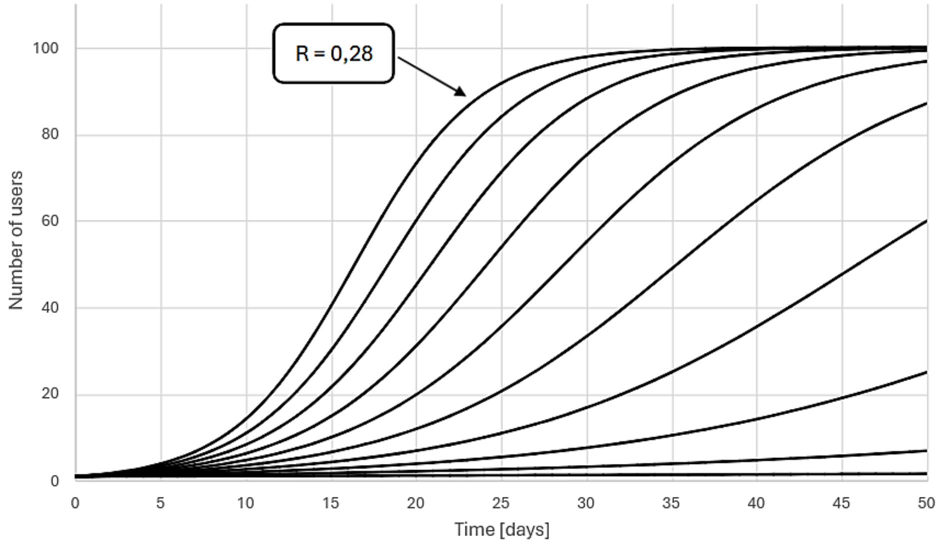

Figure 3a. Cumulative distribution curve of the logistic curve model under varying input parameters (N = 100, Np = 1, r ∈ <0.012; 0.28> step 0.03).

Source: authors' elaboration using AnyLogic Cloud.

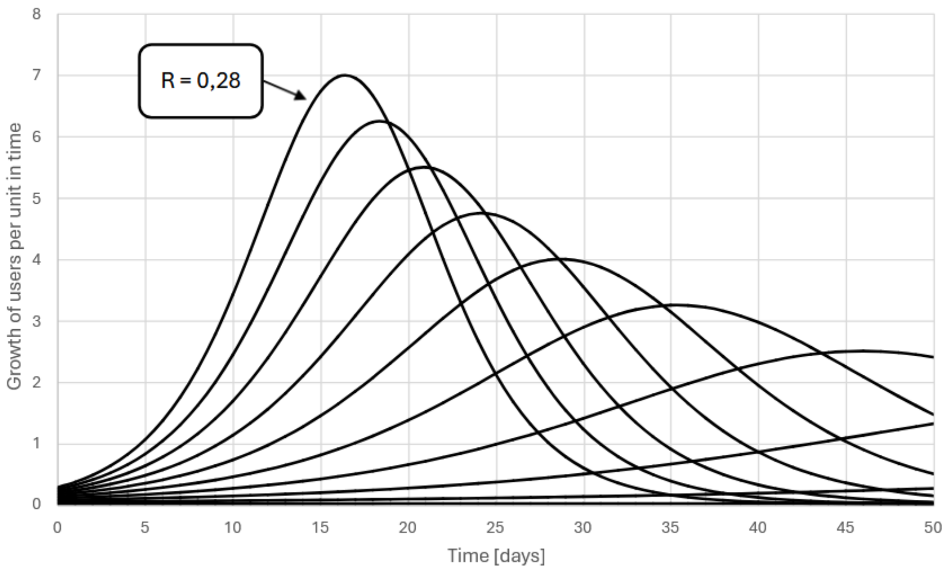

Figure 3b. Probability density curve of the logistic curve model under varying input parameters (N = 100, Np = 1, r ∈ <0.012; 0.28> step 0.03).

Source: authors' elaboration using AnyLogic Cloud.

Observing the above relationships (Figure 3a and Figure 3b), it can be clearly stated that in the logistic curve model, the higher the growth rate coefficient, the faster the diffusion of innovation occurs, and the moment when it is strongest occurs sooner.

The figures below (Figure 4a and Figure 4b) show changes in the source model of innovation diffusion in terms of changes in the proportionality coefficient.

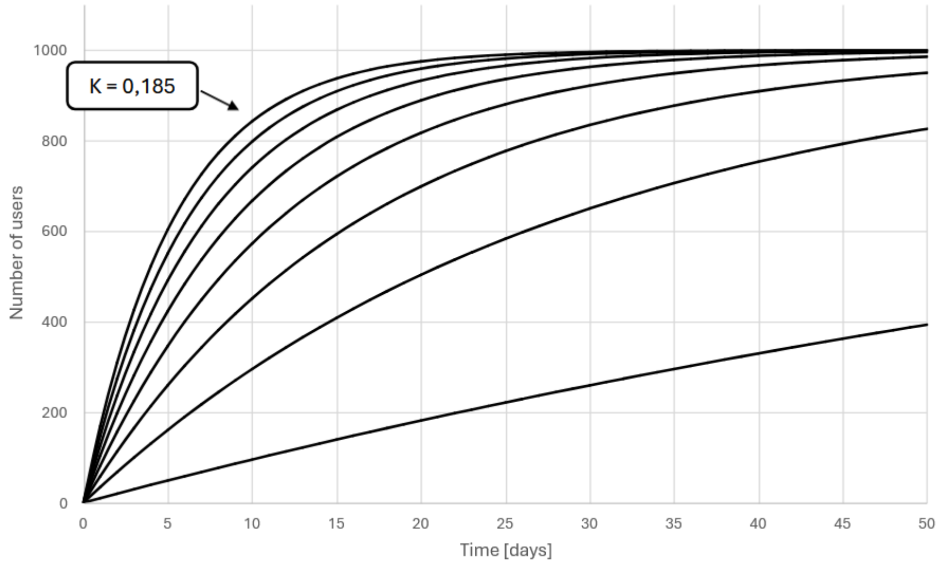

Figure 4a. Cumulative distribution curve of the source model under varying input parameters (N = 1000, Np = 1, k ∈ <0.01; 0.185> step 0.025).

Source: authors' elaboration using AnyLogic Cloud.

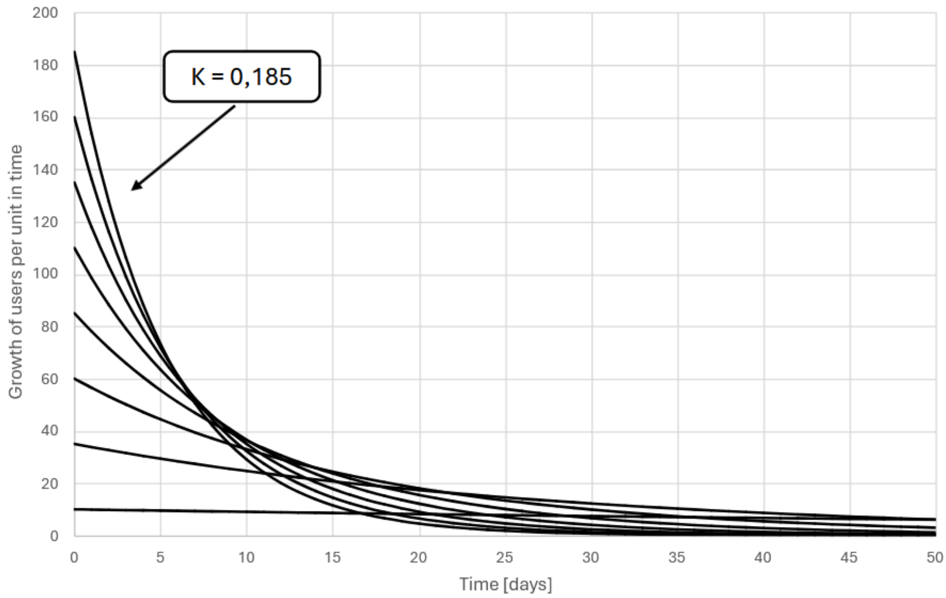

Figure 4b. Probability density curve of the source model under varying input parameters (N = 1000, Np = 1, k ∈ <0.01; 0.185> step 0.025).

Source: authors' elaboration using AnyLogic Cloud.

The source model of innovation diffusion (Figure 4a and Figure 4b) also shows that the higher the proportionality coefficient, the faster the diffusion occurs. However, a different nature of the curve can be observed, both in terms of user growth over time and maximum growth, which in this model always occurs at the very beginning of diffusion.

The figures below (Figure 5a and Figure 5b) show changes in the contact model depending on changes in the initial population size, with f and l remaining constant.

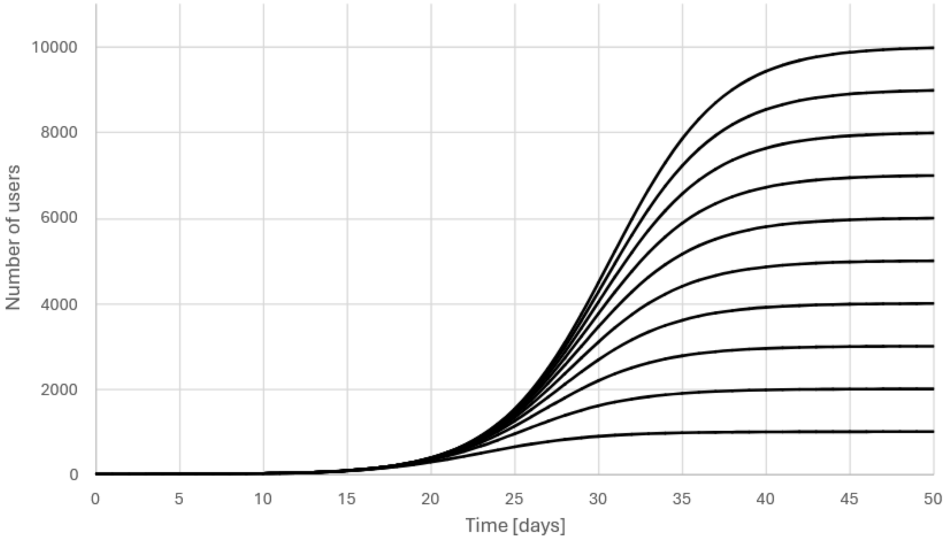

Figure 5a. Cumulative distribution curve of the contact model under varying input parameter s (N ∈ <1000; 10000> step 1000, Np = 1, f = 0.15, l = 2).

Source: authors' elaboration using AnyLogic Cloud.

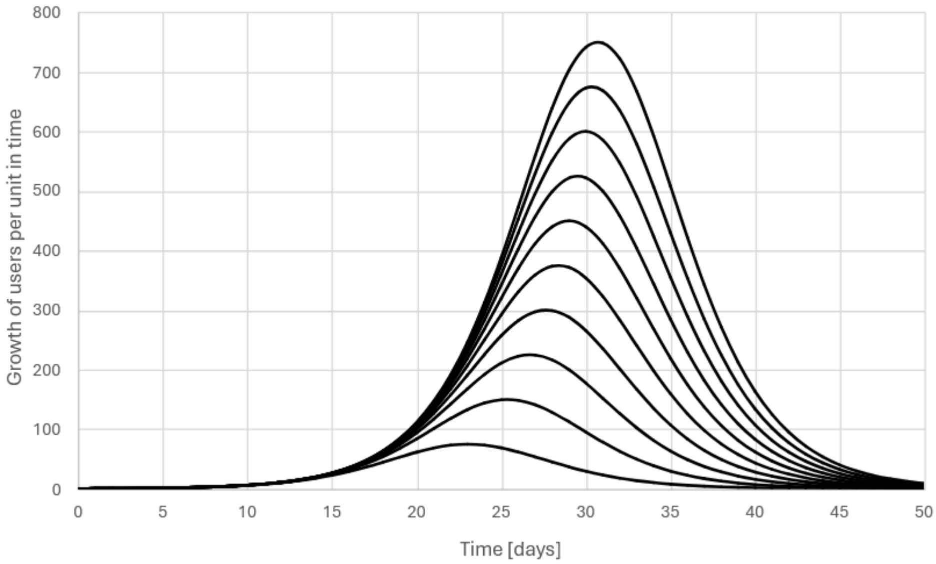

Figure 5b. Probability density curve of the contact model under varying input parameters (N ∈ <1000; 10000> step 1000, Np = 1, f = 0.15, l = 2).

Source: authors' elaboration using AnyLogic Cloud.

The figures above (Figure 5a and Figure 5b) show a different relationship. In the contact model of innovation diffusion, the size of the population in which diffusion occurs was changed. The other parameters related to the initial number of users and the coefficients responsible for the rate of diffusion remained constant. It can be observed that, despite the same coefficients, diffusion occurs more slowly as the population grows. Also, the moment when the maximum flow occurs is later in time, although its maximum is greater than in smaller populations.

The figures (Figure 6a and 6b) below show the results of simulations on a contact-source model, in which the proportionality coefficient, effectiveness of persuasion and number of contacts were changed.

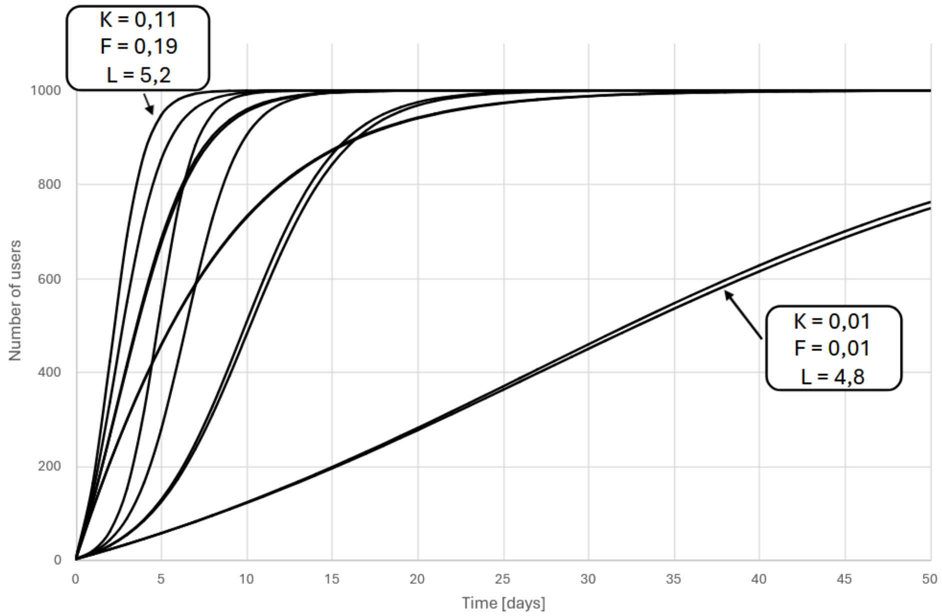

Figure 6a. Cumulative distribution curve of the contact–source model under varying input parameters (N = 1000, Np = 1, k ∈ <0.01; 0.15> step 0.1, f ∈ <0.01; 0.2> step 0.06, l ∈ <4.8; 5.2> step 0.2).

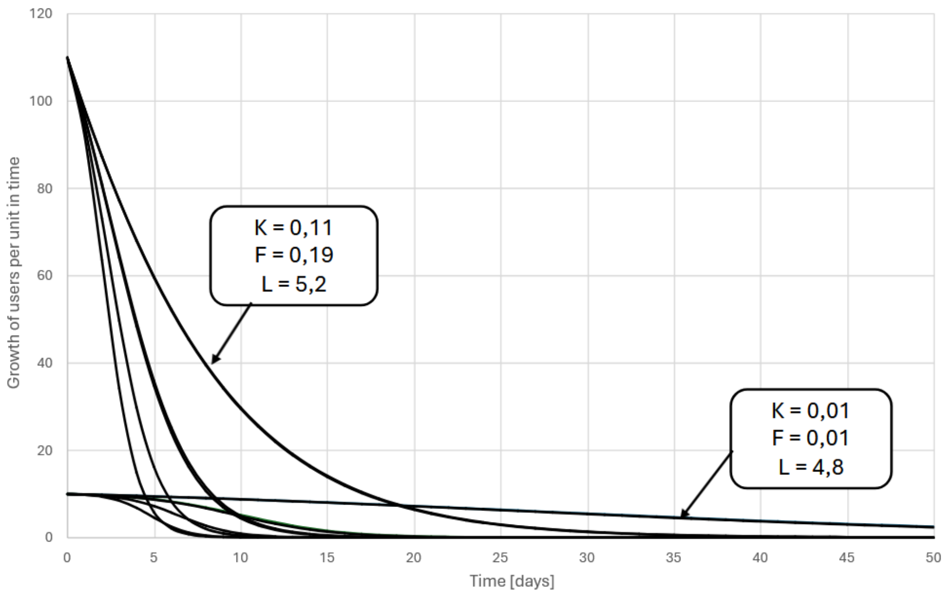

Figure 6b. Probability density curve of the contact–source model under varying input parameters (N = 1000, Np = 1, k ∈ <0.01; 0.15> step 0.1, f ∈ <0.01; 0.2> step 0.06, l ∈ <4.8; 5.2> step 0.2).

Source: authors' elaboration using AnyLogic Cloud.

Based on the results of simulations using the contact-source model, it can be seen that it is highly sensitive to changes in the parameters describing the diffusion of innovation. Depending on the initial values, this model may behave more like a contact model, a source model, or somewhere between these two characteristic curves. The curve representing the flow rate in a given unit of time behaves similarly.

Conducting a series of experiments in AnyLogic Cloud for all five developed innovation diffusion models made it possible to proceed with more detailed simulations of individual models in the context of the previously posed research questions. For this purpose, the desktop version of AnyLogic was used due to its higher computational accuracy. Selected results of the identified dependencies in the analyzed models are presented below. Some of them relate to consumer groups derived from Rogers' model of adopters, which assumes the following percentage distribution of the population: innovators 2.5%, early adopters 13.5%, early majority 34%, late majority 34%, and laggards 16% (Rogers, 1983).

Based on the simulation series, it was also possible to determine parameter values under which diffusion occurs within the same time frame as in the Bass model with specific initial conditions. These values were then used for further model comparisons in terms of flow characteristics over different time intervals. The data are summarized in Table 2.

Table 2.
Parameter values for standardized models

Model name	Population (N)	Initial users (Np)	Parameter values	Diffusion time of 98% population
The Bass model	1000	1	p = 0,01 q = 0,3	23,39
The logistic curve	1000	1	r = 0,4625	23,29
The source model	1000	1	k = 0,1675	23,29
The contact model	1000	1	f*l = 0,463	23,29
The contact-source model	1000	1	f*l $k = 0,97(f \cdot l)^2 - 0,84(f \cdot l) + 0,17$ (7)	23,29

Source: authors' elaboration.

Additionally, during the simulations it was possible to establish the relationship between the innovation coefficient (p) and the imitation coefficient (q) in the Bass diffusion model in such a way that the total diffusion time remained constant and equal to that obtained for p = 0.01 and q = 0.3. This dependency can be expressed as follows:

$$q = 0,3383e^{-12,71p} \tag{8}$$

The results obtained in this way, together with further iterations of the simulations, provide a basis for drawing conclusions about the identifying features of the individual models. The next section discusses the similarities and differences revealed during the analysis.

4. Discussion

Impact of Changes in Initial Model Parameters on Total Diffusion Time

Understanding the relationship between initial parameters and the course of diffusion for each of the discussed models is a key issue, as it may provide answers to questions regarding, for example, the profitability of financial investments in marketing, advertising, or other activities. These parameters represent factors influencing aspects of the model associated with the propagation of the innovation sponsor.

In the simulation scenarios, only the values of initial parameters were modified, while all other quantities describing the model and population remained unchanged.

For the logistic curve model, it was observed that changes in the growth rate coefficient (r) did not affect the shape of the curve; consequently, the percentage distribution of total diffusion time across Rogers' adopter categories remained constant. However, the relationship between the growth rate coefficient and the total diffusion time did change. Specifically, the higher the coefficient, the shorter the diffusion time. This relationship exhibited an exponentially decreasing character. Similar properties were observed in the contact model, where the values of the product of the persuasion effectiveness coefficient (f) and the number of contacts per person per unit of time (l) were varied.

In the source model of innovation diffusion, a mutual relationship was also noted between changes in the proportionality coefficient (k) and the diffusion time. This dependence likewise showed an exponentially decreasing character, though with different coefficients. Importantly, the impact of k on the curve's shape and on the percentage distribution of diffusion time across Rogers' adopter groups remained unchanged. Compared to the two earlier models, however, the source model demonstrated significantly faster diffusion among innovators and early adopters, and slower diffusion among laggards.

For the Bass model, simulations had to be divided into two stages due to the presence of two coefficients: the innovation coefficient (p) and the imitation coefficient (q). The simulation series revealed that the model behaved differently depending on which parameter was varied. With an increase in p, the total diffusion time decreased across all Rogers' adopter groups. When examining the percentage share of these times within total diffusion, a relationship became evident: as q increased, the percentage of diffusion time among innovators and early adopters grew, while that among the early and late majority and laggards decreased. This finding supports the theoretical interpretation of the q parameter, which describes the degree of information transfer within the population. Stronger influence leads to greater effectiveness in the early groups, while later groups are less affected. Conversely, when modeling changes in q, greater information exchange within the population resulted in faster adoption by later groups of customers, but adoption in the initial groups (without marketing interventions) was very slow.

For both dependencies, the changes in diffusion time exhibited a power-law character; however, for q, the magnitude of this effect was much smaller than for p (with regression values ranging from –0.149 to –0.819). Similar analogies in terms of variability were also observed in the contact–source diffusion model. The only difference was that the relationship between k and total diffusion time followed a logarithmically decreasing pattern.

Impact of Population Size on Total Diffusion Time

Scenarios involving changes in the size of the population within which diffusion occurs are of limited significance for companies aiming to sell innovative goods or services on the market, since they typically assume the largest possible target group of consumers. The situation is different for firms introducing process or organizational innovations, where a key question arises: should these innovations be implemented simultaneously across the entire workforce, or initially focused on a specific department, division, or process?

In the analyzed scenarios, only the population size (N) was varied, while all other model parameters remained constant throughout the simulation process.

As in the first scenario, similar relationships were observed between the logistic curve model and the contact model. In both cases, the relationship between population size and total diffusion time can be described as logarithmically increasing. This means that the larger the population, the slower the diffusion process. However, as population size grows further, the differences become progressively smaller. Observing the behavior of these models within Rogers' adopter categories revealed that the percentage share of innovators increased as the population expanded. This indicates that the larger the group in which diffusion takes place, the more time is required to reach the initial 2.5% of adopters in the logistic and contact models. Once this threshold is achieved, however, diffusion begins to progress more dynamically, making it easier to reach subsequent consumer groups.

In the source model, an interesting relationship was identified. Simulation results showed that population size had no effect on whether diffusion time lengthened or shortened. This means that regardless of how large the population is, if each individual has the same probability of adopting the technology, the total diffusion time remains unchanged. However, it should be noted that this condition assumes a constant value of the coefficient k. In practice, this implies that marketing expenditures remain fixed over time in order to achieve the same level of effectiveness.

When examining the results from the Bass model and the contact–source model, no clear mathematical relationship could be established between population size and diffusion time. For smaller population sizes, increases in diffusion time were observed, whereas for larger populations the diffusion time tended to stabilize at a constant level—though at different values for each of the two models. This effect stems from the fact that these models combine two approaches to innovation diffusion: in one, diffusion time is independent of population size, while in the other it is described as logarithmically increasing.

Impact of the Initial Number of Users on Total Diffusion Time

An important relationship to consider when designing processes associated with innovation diffusion is the initial number of users already adopting a given technology. Their presence can trigger the so-called "penguin effect", through which additional potential users are encouraged to adopt the technology by following their example (Klincewicz, 2011). This factor is particularly relevant when launching product distribution in consumer markets, but also within enterprises, where early adopters among employees may encourage others to embrace change.

In the simulation scenarios, only the initial number of users at the start of diffusion was varied, while all other model parameters were kept constant.

Both the logistic curve and contact models are characterized by the necessity of having at least one initial user of the technology; otherwise, diffusion will never occur. The relationship between changes in the number of initial users and total diffusion time can be described as logarithmically decreasing. The percentage share of total diffusion time attributed to innovators and early adopters decreases substantially, while the shares for the early majority, late majority, and laggards increase correspondingly. However, the actual diffusion time within each group decreases—only the relative proportions of time change.

For the source model, a linearly decreasing relationship was identified, though with a very low slope (–0.01). Thus, the relationship can be considered practically constant, with only a minimal reduction in total diffusion time, attributable to the smaller number of individuals who must undergo diffusion during the simulation. Differences across Rogers' adopter groups are negligible.

Both the Bass model and the contact–source model proved challenging to describe in terms of a precise relationship between the initial number of users and diffusion time. It can, however, be definitively stated that such a relationship exists and has a positive effect on reducing total diffusion time. Nevertheless, the exact nature of this dependence is difficult to determine. Both parabolic and exponential forms of the relationship yielded similar regression coefficients of approximately 0.98. In the simulations, reductions were observed in the percentage share of diffusion time among innovators and early adopters, followed by an increase among the early and late majority, and then another decrease among laggards.

Comparison of Diffusion Times across Rogers' Adopter Categories in Averaged Models

The simulation results for the logistic curve and the contact model are very similar. This arises from the comparable structure of their parameters and their influence on flow magnitude. In the contact model it was additionally demonstrated that the combined product of parameters f and l does not affect the shape of the cumulative distribution curve. Compared with the reference Bass model (with $p = 0.01$ and $q = 0.3$), both models exhibit a slow initial increase within the innovator group. Subsequently, this difference diminishes to the point that, within the laggard group, innovation spreads more rapidly.

The source model is characterized by a highly dynamic initial increase in the number of users. The innovator group requires only 0.68% of the total diffusion time. Subsequent groups are persuaded at an increasingly slower pace, to the extent that laggards account for over 53% of total diffusion time (compared to approximately 30% in the baseline Bass model). Relative to the Bass model, the source model demonstrates much faster diffusion among innovators, early adopters, and the early majority, a similar pace in the late majority, and significantly slower diffusion among laggards.

From the analysis of results for the Bass and contact–source models, it follows that even though the total diffusion time across the entire population is identical, the behavior of each model in the tested scenarios differs and depends on the parameter combinations. When the parameters associated with the contact-based approach (i.e., the product f × l and parameter q) dominate, the results resemble those of the logistic curve and the contact model. Conversely, when the parameters associated with the sponsor's influence (k and p) prevail, the outcomes resemble those of the source model.

This indicates that these models are flexible in terms of parameter configurations depending on real-world conditions. This flexibility is a major advantage, as it allows for a more accurate representation of the diffusion process—not only in terms of total diffusion time, but also in relation to the conditions and the manner in which the process unfolds.

Comparison of the Moments of Maximum Flows in Averaged Models

To determine the magnitude of flows at specific points in time, flow plots were developed for each model. These represent the first derivatives of the cumulative curves previously discussed for each diffusion model. In this analysis, the parameters specified in Table 1 were used, while the remaining model characteristics were held constant.

For the logistic curve, source, and contact models, only a single simulation was conducted, since earlier analyses had already identified parameter conditions under which these models could be made comparable to the Bass model with p = 0.01 and q = 0.3. In both the logistic and contact models, the curves are distributed symmetrically. The maximum flow values occur at the boundary between the early majority and late majority. The other adopter groups display similar distributions due to the symmetry of these curves.

In the source model, the maximum flow occurs right at the beginning of the diffusion process, within the innovator group. The rate then gradually decreases as the population becomes saturated. Compared with the logistic curve and the contact model, the maximum flow in the source model is considerably higher (by approximately 30%).

The Bass and contact–source models, due to their universal character and adaptability to different diffusion conditions, can exhibit maximum flows at varying points. For p = 0.01 and q = 0.3, the peak occurs symmetrically at the midpoint of diffusion—at the boundary between the early and late majority. However, with different parameter combinations, the peak may shift to earlier adopter categories, such as early adopters or even innovators.

Based on the earlier simulation results, the following quantitative conclusions can be drawn regarding the analyzed diffusion models:

- Increasing initial parameter values leads to a decrease in total diffusion time for a fixed population size. This relationship is power-law for the logistic, Bass, source, and contact models, and logarithmic for the contact–source model.
- In the logistic, source, and contact models, changes in initial parameters do not alter the percentage distribution of diffusion time across Rogers' adopter categories—the curves retain their shape, and diffusion proceeds proportionally over time.
- In the Bass and contact–source models, the proportions of diffusion time across Rogers' adopter categories vary depending on the relative strength of parameters, which causes the shape of the curve to change with parameter adjustments.
- Changes in population size do not affect diffusion in the source model. With a constant k, the total diffusion time remains unchanged, as does the time distribution across Rogers' categories.
- In the other models, population size influences total diffusion time. For the logistic and contact models, the dependence is logarithmically increasing; for the Bass and contact–source models, the relationship is harder to characterize, though in general, larger populations are associated with longer diffusion times.
- In these latter models, increasing population size also shifts the percentage distribution of diffusion time: the shares for innovators and early adopters grow, while those for the early and late majority, as well as laggards, shrink.
- When the initial number of users increases, total diffusion time decreases across all models. The logistic and contact models exhibit logarithmic decline, the source model shows linear decline, while the Bass and contact–source models demonstrate mixed effects from overlapping dependencies.
- With more initial users, the share of diffusion time for innovators and early adopters declines, while it increases for the other groups—except in the Bass model, where laggards once again show a decrease.
- In scenarios where parameter adjustments allowed the models to produce identical total diffusion times, it was demonstrated that the models can be synchronized for further comparison and simulation.
- It is possible to establish interdependencies between parameters in the Bass and contact–source models such that the total diffusion time remains constant under specific conditions.
- Simulations under equal total diffusion times enable interpretation of individual model properties. For the logistic and contact models, diffusion times in Rogers' groups are similar—initially low, accelerating in later groups, and slowing again among laggards.
- The source model is marked by rapid initial growth that slows sharply over time, with laggards requiring over 50% of total diffusion time.

- In the Bass and contact–source models, even when parameters are adjusted to yield identical total diffusion times, diffusion durations across adopter categories differ. Depending on parameter dominance, the models may resemble either the source model or the logistic/contact models.
- Simulations of flow dynamics reveal where models are most effective. The logistic and contact models show symmetric curves, with the peak at the midpoint (between early and late majority).
- The source model peaks at the very beginning (innovators), with diffusion rates declining steadily thereafter, leading to substantially different timing.
- In the Bass and contact–source models, peak flow positions vary with parameter combinations. Greater influence of source-related parameters shifts the peak to innovators or early adopters, while stronger influence of interaction-related parameters shifts the peak toward the early majority, resembling the symmetric logistic/contact pattern.

In addition to these quantitative findings, the analysis of causal loop diagrams (CLDs) and stock–flow diagrams (SFDs) also provide the following qualitative conclusions:

- There are structural similarities between the logistic and contact models, though they differ in how flows are described and in the number of defining parameters.
- The Bass and contact–source models also share similarities but differ in how flow magnitudes are formulated.
- In the logistic and contact models, emphasis is placed on internal interactions within the population, i.e., between individuals and the current number of users.
- In the source model, emphasis is placed on the strength of the sponsor's influence in converting potential users into adopters.
- The Bass and contact–source models appear to represent a hybrid of the other models, combining aspects of both the source-driven and contact-driven approaches.

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