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INVESTMENT RISK ASSESSMENT IN THE RAW MATERIALS MARKET IN THE ERA OF CHANGES

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Purpose: Dynamic changes taking place in investment markets, related to the negative effects of the COVID-19 pandemic and armed conflicts in the world, have led to an intensification of research related to the search for effective risk management methods. In the case of the raw materials market, investment risk is often identified with the volatility of their prices. One of the alternative measures of volatility, and therefore risk, is the fractal dimension. The aim of the research will be to assess investment risk in the raw materials market using the fractal dimension.
Design/methodology/approach: Investment risk assessment was performed based on value at risk and a non-classical risk measure, which is the fractal dimension. Rescaled range analysis based on the Hurst exponent was used to estimate this dimension.
Findings: The paper attempts to assess investment risk in the raw materials market using a non-classical method, which is the fractal dimension. It has been shown that raw materials, especially metals, can be a significant source of capital multiplication in the event of economic and financial crises, because they are characterized by lower risk.
Research limitations/implications: The use of different methods for estimating the fractal dimension gives similar but different results. Any discrepancies do not result only from possible imperfections of the methods used, but are the effect of applying them to series of finite length, i.e. only for a sufficiently large number of observations will the values of the fractal dimension approach theoretical levels.
Practical implications: The modern investment market is very competitive, therefore risk analysis and assessment is extremely important for every investor.
Originality/value: Study of the influence of anomalies (crisis phenomena) on the value of the fractal dimension.
Keywords: Investment risk, raw materials market, fractal dimension, rescaled range analysis.
Category of the paper: Research paper.

1. Introduction

From an investment perspective, the raw materials market is an interesting alternative to investing in the capital market, especially during periods of economic crisis. An example is the above-average increase in the price of gold in mid-2008, which coincided with huge declines in the financial markets. Therefore, the dynamic changes currently taking place on the capital market, related to the negative effects of the Covid-19 pandemic and the war in Ukraine, have increased interest in investments that will diversify the portfolio and led to an intensification of studies on the search for effective risk management methods (Drozd, 2020; Krężołek, 2020).

In the case of raw materials, investment risk is often identified with the volatility of their prices. One of the alternative measures of volatility, and therefore risk, is the fractal dimension. It determines the degree of jaggedness of the time series graph, which allows us to assume that the larger the dimension of the series, the greater its volatility. In this case, financial instruments whose return rate series have a larger dimension are more volatile, which means that they are more risky.

The fractal dimension was created as a measure of geometric objects. However, the scope of applications has naturally expanded, including the description of time series. Currently, this dimension is used in medicine (Beckers et al., 2006; Sobolewska-Siemieniuk et al., 2007; Lawrence et al., 2015), in urban planning (Chen, 2013; Wang, 2017; Jahanmiri, Parker, 2022), or economics and finance (Mandelbrot, 2010; Orzeszko, 2010; Sanchez-Granero et al., 2012; Andronache et al., 2016). The methodology for estimating the fractal dimension is also being developed (Zwolankowska, 2000; Sy-Sang, Feng-Yuan, 2009, Przekota, 2003). In Polish literature on the subject, much attention was devoted to the fractal dimension by Buła (2017), Zeug-Żebro (2020), Przekota (2022).

The aim of the article was to examine investment risk during the changes observed in the contemporary world and to apply the fractal dimension to assess the volatility of return rates of selected commodities. The study was conducted based on value at risk and a non-classical risk measure, which is the fractal dimension. Rescaled range analysis was used to estimate this dimension. The study used time series created from the closing prices of futures contracts for commodities: aluminum, natural gas, crude oil and gold. The data covered the period from 01.01.2018 to 31.10.2023.

2. Changes in raw material prices in the face of the challenge s of the modern world

Raw materials are the subject of numerous market transactions. They play a major role in various investment strategies, and are also widely used in industry. Their prices are fundamentally influenced by the relationship between supply and demand (in relation to their use), on the one hand, and by the investment demand for these assets, on the other. The beginning of the 21st century brought significant changes to the raw materials market. Investors have begun to treat them as alternative and safe investment assets. As a result, raw materials (derivatives) are increasingly used to diversify investment portfolios.

Commodity prices are also largely dependent on economic, political and climatic events around the world. Floods, droughts, armed conflicts or recessions can cause significant fluctuations in the price of these assets. Good examples include the coronavirus pandemic (the rise in gold prices and the fall in aluminum, natural gas and oil prices) and the war in Ukraine and the pressure it has caused on the commodity and energy markets.

Price changes in the raw materials market are usually long-term. They could be observed in the case of the raw materials already mentioned in the article (aluminum, crude oil and natural gas). Their stock prices increased significantly between 2021 and 2022 along with the economic recovery after the coronavirus pandemic (Figure 1).

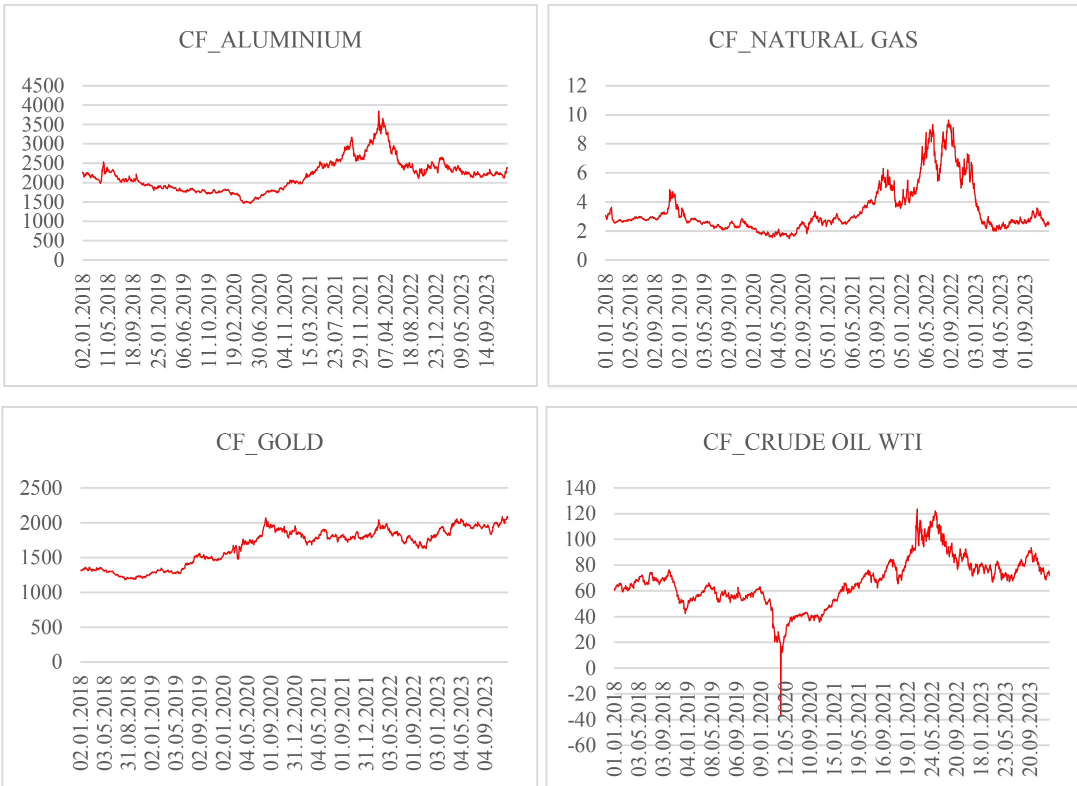

Figure 1. Price quotations for contracts for aluminum, gold, natural gas and WTI oil in the period from 1 January 2018 to 31 December 2023.

Source: own study based on stooq.pl.

In 2018-2019, the average futures prices for aluminum, crude oil, natural gas and gold were quite stable. The values of the coefficients of variation for this period were low (Table 1).

The outbreak of the COVID-19 pandemic and the restrictions introduced in many countries led to a collapse in global demand for many goods. In the first months of the crisis, significant declines in quotations could be observed among the analyzed commodities. Taking into account the entire period of 2020-2021, a decrease in the average price could be observed only for crude oil ($53.72/bbl). Other commodities recorded an increase, with the highest quotations being achieved by gold contracts ($1789.09/ozt). In 2020-2021, the calculated coefficient of variation was at the level of: 0.33 for crude oil and 0.38 for natural gas (increasing three times compared to the previous sub-period of analysis). The lowest value of this indicator of 0.06 could be observed for gold.

Table 1.
Characteristics of price volatility of futures contracts for selected raw materials in 2018-2023

Descriptive statistics	Crude oil WTI	Aluminium	Natural gas	Gold
	2018-2019			
Average price	60.98	1961.43	2.80	1334.43
Minimum price	42.53	1705.00	2.07	1184.00
Maximum price,	76.41	2537.00	4.84	1560.40
Standard deviation	6.68	181.93	0.48	99.43
Coefficient of variation	0.11	0.09	0.17	0.07
	2020-2021			
Average price	53.72	2108.53	2.93	1789.09
Minimum price	-37.63	1462.00	1.48	1477.90
Maximum price,	84.65	3171.50	6.31	2069.40
Standard deviation	17.48	443.21	1.13	106.24
Coefficient of variation	0.33	0.21	0.38	0.06
	2022-2023			
Average price	86.03	2501.46	4.60	1880.25
Minimum price	66.74	2114.00	1.99	1630.90
Maximum price,	123.70	3849.00	9.65	2093.10
Standard deviation	12.85	381.84	2.24	107.21
Coefficient of variation	0.15	0.15	0.49	0.06

Source: own study.

In the first months of 2022, as tensions on the Russian-Ukrainian border intensified, the prices of all the analyzed raw materials began to rise. In the second quarter of this year, the maximum prices of these raw materials reached very high levels: crude oil $123.70/bbl, aluminum $3849/t, natural gas $9.65/mmBtu and gold $1880.25/ozt. Despite some declines in the prices of these raw materials at the end of the third quarter of 2022, their prices remained at very high levels. The persistence of such high prices of crude oil or cold gas had a negative impact on global economic growth and caused an increase in inflation in many countries. In 2023, the prices of the raw materials under consideration (except gold) began to decline and returned to a similar level from before the pandemic. The coefficients of variation in the period 2022-2023 for crude oil, aluminum and gold took low values (≤ 0.15). The exception was the result obtained for natural gas equal to 0.49, for which high volatility of quotations can be observed.

3. Selected risk measures

The fractal dimension is one of the characteristics of chaotic systems and is used to describe structurally complex geometric objects, e.g. time series. This dimension examines the extent to which the analyzed object (series) fills the space in which it is embedding (Orzeszko, 2010). Its characteristic feature is the fact that it can take non-integer values, e.g. a curve on a plane has a dimension from the interval [1, 2].

The relationship between the variability of a series and its fractal dimension can be easily observed on a graph. Series characterized by greater variability have more jagged graphs, and consequently a larger dimension. This relationship is clearly illustrated in Figure 2. It shows three graphs of the Weierstrass-Mandelbrot function for different values of the fractal dimension.

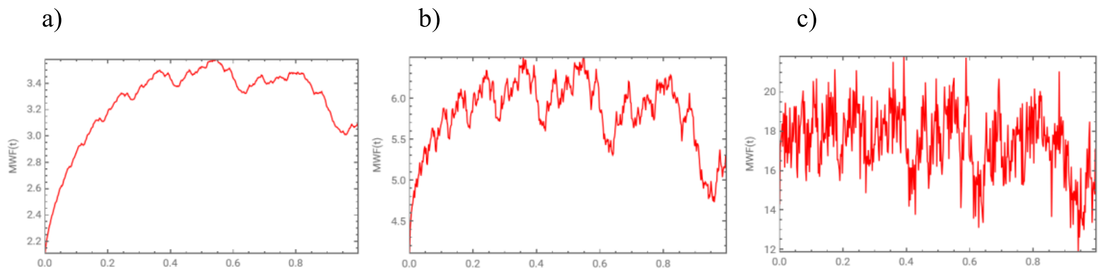

Figure 2. Weierstrass-Mandelbrot function for different values of fractal dimension: a) $D(A) = 1.1$, b) $D(A) = 1.5$, c) $D(A) = 1.9$.

Source: own study.

It has been proven that the graphs of the Weierstrass-Mandelbrot function are fractals (Dubuc et al., 1989).

In the case of fast-changing series (anti-persistent, $D(A) > 1.5$), the higher the fractal dimension, the more often a trend reversal can be observed. In turn, for slow-changing series (persistent, $D(A) < 1.5$)), the lower the value of this dimension, the stronger the trend strengthening phenomenon. For this reason, the fractal dimension has been recognized as an important characteristic of time series from the financial market, allowing for the assessment of investment risk (Buła, 2012).

The fractal dimension of a given geometric object A can be calculated by estimating the minimum number of closed hypercubes with side length ε needed to cover it. This dimension is determined based on the following formula:

$$D(A) = \lim_{\varepsilon \to \infty} \frac{\ln L(A, \varepsilon)}{\ln\left(\frac{1}{\varepsilon}\right)}, \tag{1}$$

where $L(A, \varepsilon)$ is the minimum number of hypercubes with side length ε.

A significant problem related to the study of financial time series in the fractal approach is the fact that their graphical representations should be classified as natural stochastic fractals. Due to the impossibility of a priori determining the laws governing the fluctuations of the quantities studied, it is also impossible to calculate the fractal dimension and it is necessary to use appropriate estimation methods:

- segmental-variational method (Zwolankowska, 2000),
- field division method (Przekota, 2003),
- rescaled range method (Hurst, 1951; Peters, 1994; Kale, Butar Butar, 2011).

One of the procedures allowing the calculation of the fractal dimension $D(A)$ of the time series is the analysis of the rescaled range or, briefly, the R/S analysis (Hurst, 1951). This method first of all involves estimating the value of the Hurst exponent H^* (Chun et al., 2002) and then determining the fractal dimension according to the formula (Zwolankowska, 2000):

$$D(A) = 2 - H^*. \quad (2)$$

Another risk measure mentioned in the literature (Jajuga, 2007; Maginn et al., 2007; Bacon, 2008) is Value at Risk (VaR), which measures the largest expected loss in a given period for a given tolerance level. The determination of Value at Risk can be presented by the following formula:

$$P(W \leq W_0 - VaR) = \alpha, \quad (3)$$

where:
W – value of the financial instrument at the end of the period under consideration, defined as a random variable,
W_0 – current value of the financial instrument,
α – tolerance level.

From the equation (3) it follows that the probability of realizing a loss of a financial instrument equal to or greater than the VaR value is equal to the tolerance level α.

The value at risk depends on two parameters: the tolerance level and the period. The lower the tolerance level, the higher the value at risk, and if the longer the period, than the higher the value at risk (Jajuga, 2007). In addition to determining the above two parameters, it is necessary to select a method for modeling the probability distribution (Maginn et al., 2007). In the literature, there are three basic methods for estimating the quantile of the distribution: the variance-covariance method, the historical simulation method, and the Monte Carlo simulation method. The VaR method has become a standard in risk assessment. However, its disadvantages include: the difficulty of estimating the distribution of return rates and the lack of a preferred method for their estimation, frequent underestimation of the size and frequency of extreme negative return rates, failure to include positive return rates in the risk profile (Jajuga, 2007; Maginn et al., 2007).

4. Empirical analysis

The study analyzed the closing prices of futures contracts for commodities: WTI crude oil, natural gas, aluminum and gold. For this purpose, daily time series were prepared covering the period from the beginning of 2014 to December 2023. In order to check the level of investment risk depending on crisis situations, the entire period was divided into three separate sub-periods: Period 1 - before the Covid-19 pandemic (1 January 2018 - 28 February 2020), Period 2 - during the pandemic but before the armed conflict in Ukraine (01 March 2020 - 31 January 2022) and Period 3 - from the outbreak of the war to the end of 2023 (01 February 2022 - 31 December 2023). The entire six-year period was divided in this way to check how crisis conditions affect the level of investment risk.

The analysis of the time series of closing prices of futures contracts for the above-mentioned raw materials was carried out in the following stages:

1. Transformation of the time series into logarithms of the rates of return (Figure 3) according to the formula:

$$r_t = \ln(p_t/p_{t-1}), \tag{4}$$

where p_t is the price at time.

2. Estimation of Value at Risk (VaR) for rates of return.
3. Estimation of the fractal dimension for time series of rates of return based on the rescaled range analysis (R/S).
4. Analysis of the obtained results.

The logarithm of the rate of return is considered a measure of the profitability of an investment because it shows the investor how his investment is performing and thus allows him to control its course.

Figure 3. Logarithms of rates of return: aluminum, gold, natural gas and WTI oil in the period from 1 January 2018 to 31 December 2023.

Source: own study.

A frequently used measure of investment risk is Value at Risk (VaR). The higher the value of this measure, the higher the risk associated with a given financial instrument. Table 2 contains VaR values estimated for the logarithms of the rates of return of selected commodities over the last five years and for the three sub-periods described earlier.

Table 2.
Value at Risk Estimation Results – 0.95 Quantile

Series	**VaR**			
	Entire period	**Period 1**	**Period 2**	**Period 3**
Aluminium	0.022507131	0.01932597	0.020430061	0.027234908
Natural gas	0.064175002	0.045734167	0.065951185	0.078482455
gold	0.015648779	0.011976646	0.019265717	0.015233795
Crude oil WTI	0.051921685	0.032811538	0.072959545	0.043075583

Source: own study.

The results presented in Table 2 show that the highest VaR value, and thus the highest risk in the entire period, was characteristic of natural gas rates of return. Similar conclusions can be reached when considering individual sub-periods, where the value of this measure for this commodity was the highest (except for Period 2 (the pandemic period), where its value was slightly lower than the value obtained for crude oil). Since the outbreak of the war in Ukraine, VaR has been almost twice as high as in the period before the pandemic.

The next most risky raw material was crude oil. The highest VaR value for this commodity's rates of return could be observed during the Covid-19 pandemic. However, the value of this measure decreased in the third period and reached a level comparable to the period before the pandemic.

The least risky raw material turned out to be gold, for which the value at risk in the entire period and for individual sub-periods took the lowest values. The last commodity considered is aluminum, which was characterized by VaR values similar to gold throughout the period.

In the next step of the research, the fractal dimension was estimated using scaled range analysis. The obtained values are presented in Table 3.

Table 3.
Fractal dimension estimation results based on rescaled range analysis for selected commodity rates of return

Series	Fractal dimension			
	Entire period	Period 1	Period 2	Period 3
Aluminium	1.395234	1.510031	1.439266	1.420508
Natural gas	1.449525	1.419742	1.503644	1.427421
gold	1.383189	1.407909	1.425056	1.385273
Crude oil WTI	1.461215	1.435189	1.445621	1.540877

Source: own study.

Analyzing the results from Table 3, similar conclusions can be drawn as for value at risk. The riskiest investments were those related to crude oil and natural gas. The fractal dimension values for the entire six-year period for both commodities were the highest. However, it is worth noting that in the case of division into sub-periods, the value of the dimension estimated for crude oil assumed higher and higher values from period to period, while for natural gas the value of the fractal dimension increased during the pandemic, but decreased in the period related to the outbreak of the war in Ukraine. The opposite situation can be observed in the case of the data from Table 2, i.e. VaR values increased from period to period for natural gas, while in the case of crude oil they initially increased during the pandemic but decreased in the 3rd period.

Once again, gold had the lowest risk level, followed by aluminum, even though in the pre-pandemic period the fractal dimension estimated for aluminum was the highest.

5. Summary

The presented research results on investment risk assessment indicate a certain similarity in the interpretation of the results obtained on the basis of the fractal dimension and value at risk.

The most risky investments in the entire period (1 January 2018-31 December 2023) were futures contracts for natural gas and WTI oil. The least volatile were the financial series of logarithms of the rates of return determined for gold futures contracts. This fact confirmed the general opinion that investments in gold are perceived as one of the safest in difficult periods.

The values of the measures determined for the sub-periods: before the Covid-19 pandemic, during it and after the outbreak of the war in Ukraine, confirmed the earlier conclusion about the most and least risky investments. These results reflect the fluctuations in commodity prices observed on their charts (a drop in WTI oil prices during the pandemic, followed by an increase in the prices of this commodity and natural gas during the armed conflict in Ukraine). This confirms the impact of crisis conditions on investment risk.

Fractal analysis used to assess investment risk can be an important complement to classical measures, but the research and considerations undertaken in this work do not exhaust all the issues related to the risk and efficiency of investments in the raw materials market and require further work.

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