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ANALYSIS AND FORECASTING OF GROSS ELECTRICITY PRODUCTION – RENEWABLE ENERGY DIRECTIVE IN POLAND

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Purpose: This article aims to analyze data on gross electricity production in Poland under the Renewable Energy Directive and to forecast its future values.
Design/methodology/approach: The work determines forecasts for the coming years regarding the volume of gross energy production from renewable sources; the work uses models for forecasting time series, such as analytical models and the Holt model.
Findings: The work determined forecasts of gross energy production from renewable sources for the following years and determined the error values involved in these forecasts.
Practical implications: conducting data analysis and forecasting its future values using models in the study of historical data, the result of which may be an important element in the context of long-term planning, allows for understanding the dynamics of changes in the energy sector in Poland, as well as indicating directions of development.
Originality/value: The article analyzes gross energy production in Poland and presents projected values for the coming years, which are extremely important for the country's energy security, investments, and environmental protection.
Keywords: energy from renewable sources, forecasting.
Category of the paper: Research paper, general review.

1. Introduction

In the face of growing challenges related to global energy demand and the desire to reduce greenhouse gas emissions, renewable energy sources play a key role in transforming the energy sector. The Renewable Energy Directive introduced by the European Union sets ambitious goals for developing and promoting energy from renewable sources, including electricity. When analyzing the issue of supporting the production of energy from renewable sources in the European Union, it is necessary to define the treaty bases enabling the adoption of legal instruments regulating issues related to the promotion of the use of renewable energy due to two issues: provisions of the European Union's environmental policy (provisions of Title

XX of the Treaty (Art. 191-193 TFEU) on the Functioning of the European Union (TFEU) concerning the natural environment, which defines the objectives, principles and actions of the European Union in the field of environmental protection) and the provisions of the European Union's energy policy (provisions of Title XXI of the Treaty on the Functioning of the European Union (TFEU) relating to energy, which define the objectives and principles of the European Union's energy policy). Each of the mentioned policies assumes that different goals are achieved, and an extremely important aspect is the control of the legal boundaries of treaties (Pobrzeżyńska, 2020).

As one of the EU members, Poland has undertaken numerous actions to increase the share of renewable energy sources in the national energy mix. The production of electricity is the foundation of the modern economy, and it also has a significant impact on Poland's energy security, which is determined by several factors, such as the presence and development of hydrocarbon deposits, existing infrastructure enabling the import/export of energy raw materials, and the existence of production capabilities enabling access to energy carriers (e.g., fuels from refineries) or the existing raw material base (Surmacz, Paszkowski, 2023). Although the importance of renewable energy is increasing (solar energy, wind energy, water energy, geothermal energy (Słodczyk, 2010), biomass, and biogas), the share of renewable energy sources in Poland still needs to be higher. For Poland, in the coming years, the energy transformation process will be a big challenge; therefore, there is a need to implement plans to reduce greenhouse gas emissions.

The production of electricity from renewable energy sources, forecasting, and analysis are crucial in terms of energy security and ineffective investment planning. A precise analysis of gross electricity production in Poland, considering historical trends and forecasting methods, is an important tool for assessing the effectiveness of current activities and bandaging future investments in the energy sector. An extremely important aspect in the production of energy from renewable sources is state aid, as well as national support systems for the production of energy from renewable sources (Pobrzeżyńska, 2020), such as green certificate systems (Soliński, 2016), auction systems, prosumer (support system for RES micro-installations), blue system, white and red certificates, subsidies and support programs (My electricity, Clean air, Agroenergy).

This article aims to analyze data on gross electricity production in Poland under the Renewable Energy Directive and to forecast its future values using models in the analysis of historical data, the result of which may be an important element in the context of long-term planning. The forecasting methods used include various analytical approaches, including exponential, polynomial, power, linear models, and the Holt model. The analysis of the obtained results allows us not only to identify the best approach to forecasting but also to understand the dynamics of changes in the energy sector in Poland and to indicate the directions of development and potential challenges for further energy transformation in Poland.

2. Data analysis and forecasting based on analytical models and the Holt model

Table 1 presents data on Gross electricity production—Renewable Energy Directive for Poland from 2004 to 2023. The unit of gross electricity production is gigawatt-hour.

Table 1.
Gross electricity production - Renewable Energy Directive for Poland

Year	Gross electricity production
2004	2936.03
2005	3621.60
2006	4284.07
2007	5087.22
2008	6543.51
2009	8508.58
2010	10199.50
2011	12752.83
2012	16855.42
2013	17028.46
2014	19858.49
2015	21978.98
2016	22435.08
2017	22537.50
2018	22839.21
2019	24965.59
2020	27681.84
2021	30858.20
2022	37190.80
2023	43789.70

Source: https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured_custom_14213015/default/table

The article aims to perform a forecast analysis regarding gross electricity production - Renewable Energy Directive in Poland based on data from 2004-2023. Table 2 below presents the data that will be analyzed. It also determines changes in gross electricity production from year to year. The most significant differences in the year-to-year increase in gross electricity production is gigawatt-hour can be observed in the final analyzed period - from 2019.

Table 2.
Gross electricity production - Renewable Energy Directive for Poland

Year	Gross electricity production	Absolute growth (absolute) for the immediately preceding period	Relative growth for the immediately preceding period
2004	2936.03	-	-
2005	3621.60	685.570	23.35%
2006	4284.07	662.477	18.29%
2007	5087.22	803.147	18.75%
2008	6543.51	1456.292	28.63%
2009	8508.58	1965.072	30.03%
2010	10199.50	1690.912	19.87%
2011	12752.83	2553.336	25.03%
2012	16855.42	4102.590	32.17%
2013	17028.46	173.041	1.03%

Cont. table 2.

2014	19858.49	2830.026	16.62%
2015	21978.98	2120.490	10.68%
2016	22435.08	456.104	2.08%
2017	22537.50	102.417	0.46%
2018	22839.21	301.712	1.34%
2019	24965.59	2126.378	9.31%
2020	27681.84	2716.251	10.88%
2021	30858.20	3176.359	11.47%
2022	37190.80	6332.600	20.52%
2023	43789.70	6598.900	17.74%

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured_custom_14213015/default/table

Below, in Figure1, the time series regarding Gross electricity production for Poland is presented. This series is characterized by an increasing trend (the data show a clear upward trend, the values increase over subsequent years) with random fluctuations, without seasonal fluctuations (no repeatable seasonal patterns are visible in the data).

2.1. The linear model

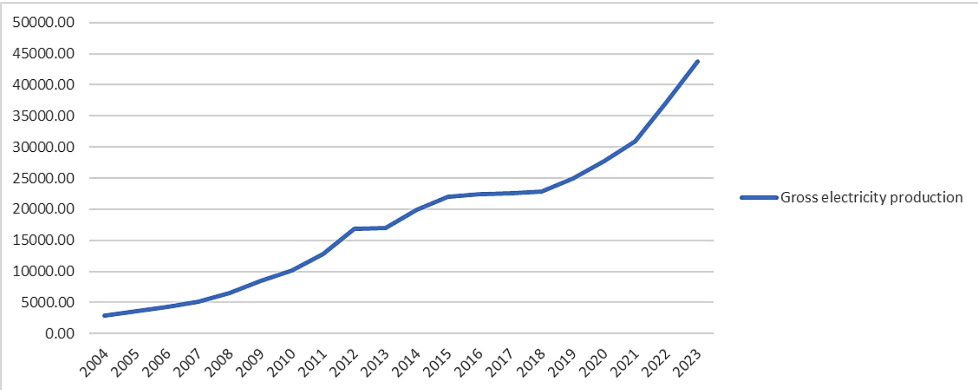

Figure 1. Gross electricity production values.

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

An analytical method was proposed for forecasting, considering the comparison of errors obtained for various trend lines and the Holt model with the determination of forecasts divided into errors used to evaluate the model. In the next steps, the analyzed series of gross electricity production values and theoretical and forecast values are presented on the charts. Based on the determined parameters for individual trend lines, forecasts for the next 5 years and errors will be determined: MAE, MAPE, RMSE, and RMSPE. The next part of the work presents a prognostic analysis using the Holt model, specifying forecasts based on errors also specified in the analytical method.

The first analysis presents a linear model of the form $\hat{y} = a + b \cdot t$, with parameters a = -827.4 and b = 1897.6, respectively. On this basis, it can be concluded that in 2004-2023, gross electricity production values increased on average from year to year by 1897.6 gigawatt-hour. The coefficient of determination was determined to be 0.9487, which means that the linear model of the form y = -1827.4 + 1897.4·t explains 94.87% of the variability of gross electricity production. The standard error of the model estimate s was 2683.488, which means that the model predictions for gross electricity production, on average, differ from the actual values by about 2683.488 gigawatt-hour.

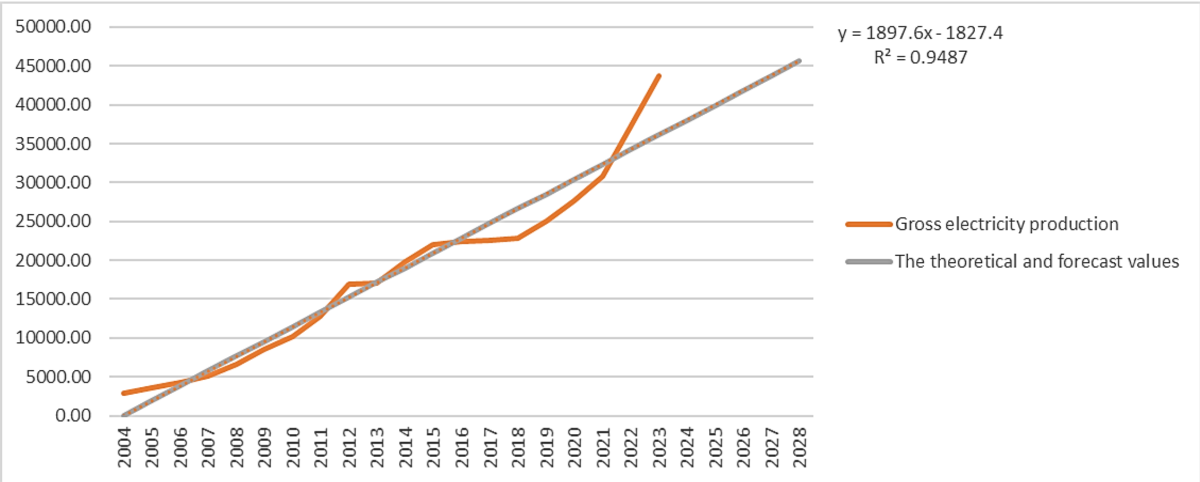

Figure 2. Gross electricity production values, along with theoretical and forecasted values for the linear model.

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the linear model defined by the formula $\hat{y}_t = -1827.4 + 1897.4 \cdot t$, the **values** of expired forecasts were determined, as presented in Table 3.

Table 3.
Real values of gross electricity production yt and values resulting for the liner model $\hat{y}$

t	Year	y_t	$\hat{y}_t$	$(t - \bar{t})^2$
1	2004	2936.03	70.24	90.25
2	2005	3621.60	1967.86	72.25
3	2006	4284.07	3865.48	56.25
4	2007	5087.22	5763.10	42.25
5	2008	6543.51	7660.72	30.25
6	2009	8508.58	9558.34	20.25
7	2010	10199.50	11455.96	12.25
8	2011	12752.83	13353.58	6.25
9	2012	16855.42	15251.20	2.25
10	2013	17028.46	17148.82	0.25
11	2014	19858.49	19046.44	0.25
12	2015	21978.98	20944.06	2.25
13	2016	22435.08	22841.68	6.25
14	2017	22537.50	24739.30	12.25
15	2018	22839.21	26636.92	20.25
16	2019	24965.59	28534.54	30.25
17	2020	27681.84	30432.16	42.25

Cont. table 3.

18	2021	30858.20	32329.78	56.25
19	2022	37190.80	34227.40	72.25
20	2023	43789.70	36125.02	90.25
Sum			**665.00**	

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the obtained trend line values for individual years, ex-ante errors (1) and relative ex-ante errors (2) will be determined for subsequent forecasts (Table 4). The patterns used are presented below (Zeliaś, Pawełek, Wanat, 2022):

- error *ante*:

$$v_t = s \cdot \sqrt{\frac{(T-\bar{t})^2}{\sum_{t=1}^{n}(t-\bar{t})^2} + \frac{1}{n} + 1} \qquad (1)$$

- relative error *ex-ante*:

$$\eta_t = \frac{v_t}{y_t^*} \cdot 100\% \qquad (2)$$

where:

s – standard error of model estimation,

T – the period for which the ex-ante error is determined,

$\bar{t}$ – average value from periods 1 to 20 or calculated from the formula (n+1)/2 = 10.5,

n – number of periods with actual values,

y_t^* – forecast determined for period *t* based on a trend line $\hat{y}_t$.

The values presented in Table 4 indicate gross electricity production forecasts for the years 2024-2028, with a relative error ranging from 6.876% for 2028 to 7.782% in 2024. These forecasts can be considered acceptable at this stage of forecasting, but they should be compared with other errors such as MAE, MAPE, RMSE, and RMSPE, which will be implemented later in the work.

Table 4.

The forecasts, ex-ante and relative ex-ante error for the linear model

Year	Forecast	*The ex-ante error*	The relative ex-ante error
2024	38022.642	2958.890	7.782%
2025	39920.262	2998.877	7.512%
2026	41817.882	3041.900	7.274%
2027	43715.502	3087.831	7.063%
2028	45613.122	3136.543	6.876%

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Interval forecasts can also be determined based on the determined point forecasts. A pattern will be used for this purpose (3):

$$[y_t^* - u \cdot v_t \,;\, y_t^* + u \cdot v_t] \qquad (3)$$

Due to differences in determining interval forecasts due to the adopted coefficient. If the hypothesis about the normal distribution of residuals was not verified or when this hypothesis was rejected, the coefficient u from Czebyszew's inequality (4) is used - used when the hypothesis about the normal distribution of residuals was not verified or when this hypothesis was rejected:

$$u = \sqrt{\frac{1}{1-p}} \tag{4}$$

where $p -$ credibility level (the probability that a random variable is within a specified range around the expected value).

An increase in the confidence level p causes u to increase, resulting in a broader range around the mean. For predictions, a larger u provides greater confidence that the value is within the range but at the cost of greater error tolerance.

When the distribution of residuals is normal, we can use Student's t-distribution tables to determine the coefficient u in order to estimate confidence intervals for the estimation. We read u for a given level of confidence *(1-α)* and *n-2* degrees of freedom (*2* - the number of parameters in a simple regression model (slope coefficient and intercept).

In order to determine interval forecasts, the Jarque-Bera test will be performed (5). This test is a statistical normality test that checks whether a sample of data comes from a normal distribution. The test is based on the analysis of two parameters: Skewness (a measure of the asymmetry of the data distribution) and kurtosis (a measure of the "pointiness" of the distribution compared to a normal distribution (Kukuła, Goryl, Jędrzejczyk, Osiewalski, Walkosz, 2009). The Jarque-Bera statistic is tested against a chi-square (χ2) distribution with 2 degrees of freedom.

- if JB > χ2 (number of degrees of freedom = 2,α) - we reject the H0 hypothesis (the distribution of residuals is not consistent with the normal distribution).
- if JB ≤ χ2 (number of degrees of freedom = 2,α) - there are no grounds to reject H0 (the distribution of residuals is consistent with the normal distribution).

The formula for the Jarque-Bera statistic:

$$JB = n \cdot \left(\frac{1}{6} \cdot B_1 + \frac{1}{24} \cdot (B_2 - 3)^2\right) \tag{5}$$

where:

$$B_1 = \left(\frac{1}{n} \cdot \frac{\sum_{t=1}^{n} e_t^3}{\bar{S}^3}\right)^2 \tag{6}$$

$$B_2 = \frac{1}{n} \cdot \frac{\sum_{t=1}^{n} e_t^4}{\bar{S}^4} \tag{7}$$

$$\bar{S} = \sqrt{\frac{1}{n} \cdot \sum_{t=1}^{n} e_t^2} \tag{8}$$

$$e_t = y_t - \widehat{y_t} \tag{9}$$

Below, in Table 5, calculations for the subsequent factors of the JB statistics are presented:

Table 5.
The partial factors for the Jarque-Bera test

t	Year	y_t	$\hat{y}_t$	$(y_t - \hat{y}_t)^2$	$(y_t - \hat{y}_t)^3$	$(y_t - \hat{y}_t)^4$
1	2004	2936.03	70.24	8212730.053	23535927745.785	67448934920702.800
2	2005	3621.60	1967.86	2734842.783	4522707984.429	7479365045343.470
3	2006	4284.07	3865.48	175220.0166	73345855.002	30702054203.241
4	2007	5087.22	5763.10	456814.0529	-308751576.173	208679078911.332
5	2008	6543.51	7660.72	1248154.414	-1394448487.123	1557889441559.910
6	2009	8508.58	9558.34	1101988.54	-1156819544.537	1214378743137.820
7	2010	10199.50	11455.96	1578703.106	-1983584450.135	2492303496546.640
8	2011	12752.83	13353.58	360898.9202	-216809532.962	130248030565.972
9	2012	16855.42	15251.20	2573525.852	4128504884.553	6623035308433.790
10	2013	17028.46	17148.82	14486.01124	-1743505.119	209844521.708
11	2014	19858.49	19046.44	659422.03	535482371.369	434837413667.445
12	2015	21978.98	20944.06	1071055.142	1108454181.254	1147159117681.450
13	2016	22435.08	22841.68	165322.0693	-67219650.345	27331386611.026
14	2017	22537.50	24739.30	4847928.849	-10674175914.013	23502414123002.200
15	2018	22839.21	26636.92	14422596.54	-54772830164.216	208011290908501.000
16	2019	24965.59	28534.54	12737414.72	-45459215201.533	162241733701275.000
17	2020	27681.84	30432.16	7564263.37	-20804149324.006	57218080326151.800
18	2021	30858.20	32329.78	2165552.702	-3186787728.259	4689618505086.570
19	2022	37190.80	34227.40	8781728.847	26023759392.655	77118761545760.500
20	2023	43789.70	36125.02	58747290.16	450279067466.276	3451244100864630.000
Sum				129619938.17	370180714802.902	4072821073856300.000

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the above data presented in Table 5, individual factors of the JB statistics were determined and presented in Table 6.

Table 6.
The partial factors for the Jarque-Bera test

e_t^2	129619938.17
e_t^3	370180714802.902
e_t^4	4072821073856300.000
$\bar{S}$	2545.780
B_1	1.258
B_2	4.848

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

The value of the JB statistic was 7.041. Comparing it with the critical value $\chi 2 = 5.991$, hypothesis H0 regarding the normality of the distribution of residuals should be rejected. Due to the inconsistency of the distribution of residuals with the normal distribution, we adopt the coefficient *u* of Czebyszew's inequality (4) to determine interval forecasts:

$$u = \sqrt{\frac{1}{1-p}} = \sqrt{\frac{1}{1-0.95}} = 4.472 \quad (10)$$

Assuming p = 0.95 means that at least 95% of the values will be in the range $\mu \pm 4.47\sigma$, where σ is the standard deviation.

Table 7.
The interval forecasts

Year	t	The point forecast	The interval forecast	
			from	to
2024	21	38022.642	24790.083	51255.201
2025	22	39920.262	26508.877	53331.648
2026	23	41817.882	28214.094	55421.670
2027	24	43715.502	29906.303	57524.702
2028	25	45613.122	31586.074	59640.171

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

With a probability of 95%, the value of gross electricity production will be in the range:

a) in 2024, from 24790.083 to 51255.201 gigawatt-hour,
b) in 2025, from 26508.877 to 53331.648 gigawatt-hour,
c) in 2026, from 28214.094 to 55421.670 gigawatt-hour,
d) in 2027, from 29906.303 to 57524.702 gigawatt-hour,
e) in 2028, from 31586.074 to 59640.171 gigawatt-hour.

In the further part of the analysis, the error values described by the formulas were determined (11) – (14) (Zeliaś, Pawełek, Wanat, 2022).

a) Mean Absolute Error

$$\boldsymbol{MAE = \frac{1}{n} \cdot \sum_{i=1}^{n} |y_t - \hat{y}_t|} \tag{11}$$

b) Mean Absolute Percentage Error

$$\boldsymbol{MAPE = \frac{1}{n} \cdot \sum_{i=1}^{n} \left| \frac{y_t - \hat{y}_t}{y_t} \right| \cdot 100\%} \tag{12}$$

c) Root Mean Squared Error

$$\boldsymbol{RMSE = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} (y_t - \hat{y}_t)^2}} \tag{13}$$

d) Root mean Squared Percentage Error

$$\boldsymbol{RMSPE = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} \left(\frac{y_t - \hat{y}_t}{y_t} \right)^2} \cdot 100\%} \tag{14}$$

Table 8 below shows the values of the proposed errors and their components.

Table 8.
MAE, MAPE, RMSE, and RMSPE error values for the linear model

t	Year	y_t	$\hat{y}_t$	MAE	MAPE	RMSE	RMSPE
1	2004	2936.03	70.24	2865.79	0.98	8212730.05	0.95273
2	2005	3621.60	1967.86	1653.74	0.46	2734842.78	0.20851
3	2006	4284.07	3865.48	418.59	0.10	175220.02	0.00955
4	2007	5087.22	5763.10	675.88	0.13	456814.05	0.01765
5	2008	6543.51	7660.72	1117.21	0.17	1248154.41	0.02915
6	2009	8508.58	9558.34	1049.76	0.12	1101988.54	0.01522
7	2010	10199.50	11455.96	1256.46	0.12	1578703.11	0.01518
8	2011	12752.83	13353.58	600.75	0.05	360898.92	0.00222
9	2012	16855.42	15251.20	1604.22	0.10	2573525.85	0.00906
10	2013	17028.46	17148.82	120.36	0.01	14486.01	0.00005
11	2014	19858.49	19046.44	812.05	0.04	659422.03	0.00167

Cont. table 8.

12	2015	21978.98	20944.06	1034.92	0.05	1071055.14	0.00222
13	2016	22435.08	22841.68	406.60	0.02	165322.07	0.00033
14	2017	22537.50	24739.30	2201.80	0.10	4847928.85	0.00954
15	2018	22839.21	26636.92	3797.71	0.17	14422596.54	0.02765
16	2019	24965.59	28534.54	3568.95	0.14	12737414.72	0.02044
17	2020	27681.84	30432.16	2750.32	0.10	7564263.37	0.00987
18	2021	30858.20	32329.78	1471.58	0.05	2165552.70	0.00227
19	2022	37190.80	34227.40	2963.40	0.08	8781728.85	0.00635
20	2023	43789.70	36125.02	7664.68	0.18	58747290.16	0.03064
Sum				38034.76	3.14	129619938.17	1.37
Error				**1901.74**	**15.72%**	**2545.78**	**26.18%**

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the calculations in the table above, the estimated errors were: MAE = 1901.74 (the linear model forecasts on average differ from the actual data by 1901.74 gigawatt-hour), MAPE = 15.72% (the model forecasts on average differ from the actual data by 15.72% of their value.). RMSE = 2545.78 - this error penalizes more significant errors more severely than MAE, which indicates that there are errors in the data that are significantly different. It is more sensitive to large differences between the actual and predicted values, resulting in more significant errors being "penalized" than smaller ones. The RMSPE error measure equals 26.18% - RMSPE emphasizes significant percentage errors more, especially at large actual values). In summary, a higher RMSE than MAE indicates more significant errors in the data. The percentage error of RMSPE (26.18%) is larger than MAPE (15.72%), which indicates a possible misfit of the model in the final, larger values of the series. A linear trend is not ideal for this data because higher values result in more significant errors, as seen in the RMSPE. It is worth considering more flexible models, such as polynomial, exponential, or power-law trends.

2.2. The polynomial model

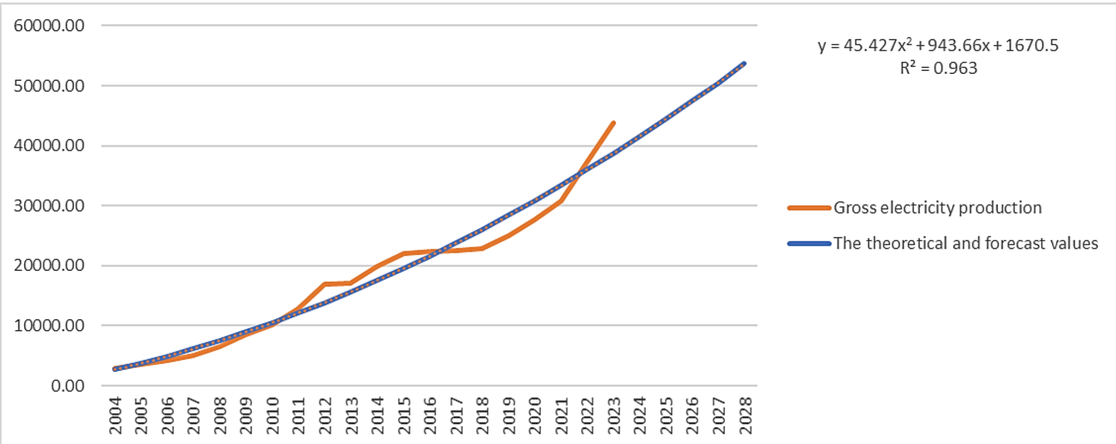

Figure 3. Gross electricity production values, along with theoretical and forecasted values for the polynomial model.

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the polynomial model defined by the formula $\hat{y_t} = 45.527 \cdot t^2 + 943.66 \cdot t + 1670.5$, forecasts for the following years 2024-2028 were calculated, respectively: 41520.49 gigawatt-hour (2024), 44417.49 gigawatt-hour (2025), 47405.35 gigawatt-hour (2026), 50484.06 gigawatt-hour (2027), 53653.62 gigawatt-hour (2028). The standard error of the model was 2343.85. In Table 9, the values of the proposed errors and their components are presented below.

Table 9.
MAE, MAPE, RMSE, and RMSPE error values for the polynomial model

t	Year	y_t	$\hat{y_t}$	MAE	MAPE	RMSE	RMSPE
1	2004	2936.03	2 659.55	276.47	0.09	76437.15	0.00887
2	2005	3621.60	3 739.50	117.90	0.03	13900.29	0.00106
3	2006	4284.07	4 910.29	626.22	0.15	392148.70	0.02137
4	2007	5087.22	6 171.94	1084.72	0.21	1176615.67	0.04546
5	2008	6543.51	7 524.44	980.93	0.15	962221.03	0.02247
6	2009	8508.58	8 967.80	459.21	0.05	210874.98	0.00291
7	2010	10199.50	10 502.00	302.51	0.03	91510.46	0.00088
8	2011	12752.83	12 127.06	625.77	0.05	391585.90	0.00241
9	2012	16855.42	13 842.98	3012.44	0.18	9074820.94	0.03194
10	2013	17028.46	15 649.74	1378.72	0.08	1900864.26	0.00656
11	2014	19858.49	17 547.36	2311.12	0.12	5341295.23	0.01354
12	2015	21978.98	19 535.84	2443.14	0.11	5968938.07	0.01236
13	2016	22435.08	21 615.16	819.92	0.04	672266.70	0.00134
14	2017	22537.50	23 785.34	1247.84	0.06	1557113.90	0.00307
15	2018	22839.21	26 046.38	3207.16	0.14	10285902.31	0.01972
16	2019	24965.59	28 398.26	3432.67	0.14	11783235.92	0.01891
17	2020	27681.84	30 841.00	3159.16	0.11	9980289.09	0.01302
18	2021	30858.20	33 374.59	2516.39	0.08	6332230.60	0.00665
19	2022	37190.80	35 999.04	1191.76	0.03	1420298.33	0.00103
20	2023	43789.70	38 714.34	5075.36	0.12	25759326.53	0.01343
Sum				34269.43	1.97	93391876.05	0.25
Error				1 713.47	9.85%	2160.92	11.11%

Source: own study based on
https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the calculations in the table above, the polynomial model predictions were estimated to differ from the actual data on average by 1713.47 gigawatt-hour (MAE). The MAPE error value was 9.85%, which means that the model's predictions, on average, differ from the actual data by 9.85% of their value. The RMSE error was set at 2160.92 - this value is greater than the MAE value, which indicates that errors in the data significantly differ (large differences between the actual and forecast values). The RMSPE error measure is 11.11%. The share of the MAE error in the forecasts was, respectively, 4.13%, 3.86%, 3.61%, 3.39%, and 3.19%, while the share of the RMSE error in the forecasts was estimated at the following levels: 5.20%, 4.87%, 4.56%, 4.28%, respectively and 4.03%.

2.3. The power model

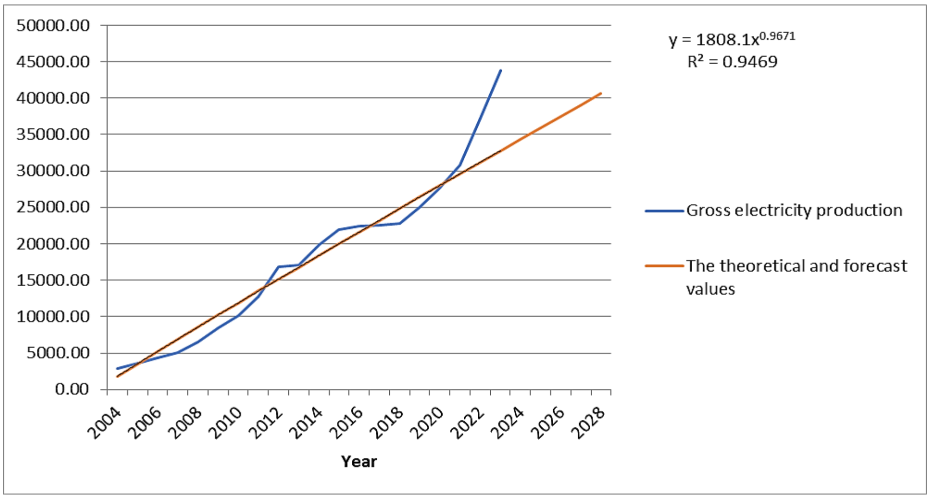

Figure 4. Gross electricity production values, along with theoretical and forecasted values for the power model.

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the power model defined by the formula $\hat{y}_t = 1808.1 \cdot t^{0.9671}$, the following forecasts were calculated for the following years 2024-2028: 34352.97 gigawatt-hour (2024), 35933.83 gigawatt-hour (2025), 37512.32 gigawatt-hour (2026), 39088.56 gigawatt-hour (2027), 40662.65 gigawatt-hour (2028). The standard error of the power model was 3261.1.

Table 10.
MAE, MAPE, RMSE, and RMSPE error values for the power model

t	Year	y_t	ln(t)	$\hat{y}_t$	MAE	MAPE	RMSE	RMSPE
1	2004	2936.03	0	1808.07	1127.96	0.38	1272292.64	0.14759
2	2005	3621.60	0.693147181	3534.66	86.94	0.02	7557.78	0.00058
3	2006	4284.07	1.098612289	5231.78	947.71	0.22	898155.59	0.04894
4	2007	5087.22	1.386294361	6910.05	1822.83	0.36	3322699.52	0.12839
5	2008	6543.51	1.609437912	8574.43	2030.91	0.31	4124608.35	0.09633
6	2009	8508.58	1.791759469	10227.82	1719.24	0.20	2955773.77	0.04083
7	2010	10199.50	1.945910149	11872.14	1672.64	0.16	2797731.16	0.02689
8	2011	12752.83	2.079441542	13508.72	755.89	0.06	571373.00	0.00351
9	2012	16855.42	2.197224577	15138.58	1716.84	0.10	2947543.73	0.01037
10	2013	17028.46	2.302585093	16762.48	265.98	0.02	70745.84	0.00024
11	2014	19858.49	2.397895273	18381.04	1477.44	0.07	2182842.66	0.00554
12	2015	21978.98	2.48490665	19994.77	1984.21	0.09	3937086.85	0.00815
13	2016	22435.08	2.564949357	21604.07	831.01	0.04	690574.87	0.00137
14	2017	22537.50	2.63905733	23209.31	671.81	0.03	451329.93	0.00089
15	2018	22839.21	2.708050201	24810.78	1971.57	0.09	3887075.71	0.00745
16	2019	24965.59	2.772588722	26408.74	1443.15	0.06	2082675.86	0.00334
17	2020	27681.84	2.833213344	28003.41	321.57	0.01	103409.69	0.00013
18	2021	30858.20	2.890371758	29595.01	1263.19	0.04	1595651.78	0.00168
19	2022	37190.80	2.944438979	31183.70	6007.10	0.16	36085284.32	0.02609
20	2023	43789.70	2.995732274	32769.64	11020.06	0.25	121441774.96	0.06333
Sum					39138.06	2.68	191426187.98	0.62
Error					1 956.90	13.41%	3093.75	17.63%

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the calculations in Table 10, an MAE error of 1956.9 was calculated, which means that the power model forecasts, on average, differ from the actual data by 1956.9 gigawatt-hour. The MAPE error value was 13.41%, which means that the model's predictions, on average, differ from the actual data by 13.41% of their value. The RMSE error value was 3093.75. The RMSPE error measure is 17.63. The share of the MAE error in the forecasts was 5.7%, 5.45%, 5.22%, 5.01%, and 4.81%, respectively, while the share of the RMSE error in the forecasts was estimated at the following levels: 9.01%, 8.61%, 8.25%, 7.91%, 7.61%.

2.4. The exponential model

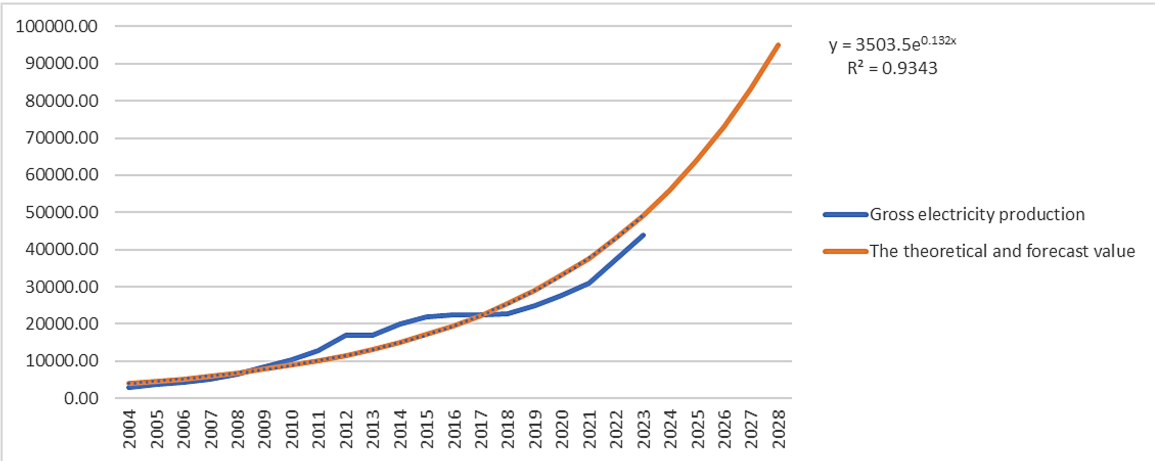

Figure 5. Gross electricity production values, along with theoretical and forecasted values for the exponential model.

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the power model defined by the formula $\hat{y}_t = 3503.5 \cdot e^{0132}$, the forecasts for the following years 2024-2028 were calculated, respectively: 55987.76 gigawatt-hour (2024), 63886.19 gigawatt-hour (2025), 72898.88 gigawatt-hour (2026), 83183.029 gigawatt-hour (2027), 94918.005 gigawatt-hour (2028). The standard error of the power model was 3896.2776. Table 11 below shows the error values and their components.

Table 11.
MAE, MAPE, RMSE, and RMSPE error values for the exponential model

t	Year	y_t	ln(y)	$\hat{y}_t$	MAE	MAPE	RMSE	RMSPE
1	2004	2936.03	7.9848	3 997.75	1061.72	0.36	1127258.46	0.13077
2	2005	3621.60	8.1947	4561.72957	940.13	0.26	883851.13	0.06739
3	2006	4284.07	8.3627	5205.271759	921.20	0.22	848607.15	0.04624
4	2007	5087.22	8.5345	5939.601124	852.38	0.17	726553.58	0.02807
5	2008	6543.51	8.7862	6777.52539	234.01	0.04	54762.27	0.00128
6	2009	8508.58	9.0488	7733.659123	774.92	0.09	600508.56	0.00829
7	2010	10199.50	9.2301	8824.678623	1374.82	0.13	1890122.82	0.01817
8	2011	12752.83	9.4535	10069.61279	2683.22	0.21	7199665.34	0.04427
9	2012	16855.42	9.7324	11490.175	5365.25	0.32	28785875.36	0.10132
10	2013	17028.46	9.7426	13111.14184	3917.32	0.23	15345405.07	0.05292
11	2014	19858.49	9.8964	14960.78522	4897.70	0.25	23987502.29	0.06083
12	2015	21978.98	9.9978	17071.3655	4907.61	0.22	24084670.26	0.04986

Cont. table 11.

13	2016	22435.08	10.0184	19479.69413	2955.39	0.13	8734323.39	0.01735
14	2017	22537.50	10.0229	22227.7757	309.72	0.01	95929.14	0.00019
15	2018	22839.21	10.0362	25363.54059	2524.33	0.11	6372234.85	0.01222
16	2019	24965.59	10.1253	28941.68089	3976.09	0.16	15809298.80	0.02536
17	2020	27681.84	10.2285	33024.60435	5342.76	0.19	28545120.21	0.03725
18	2021	30858.20	10.3372	37683.52282	6825.32	0.22	46585031.61	0.04892
19	2022	37190.80	10.5238	42999.69432	5808.89	0.16	33743253.26	0.02440
20	2023	43789.70	10.6872	49065.84028	5276.14	0.12	27837656.25	0.01452
				Sum	60948.95	3.60	273257629.80	0.79
				Error	3 047.45	18.00%	3696.33	19.87%

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the calculations in the table above, the exponential model predictions were estimated to differ from the actual data by, on average, 3047.45 gigawatt-hours (MAE). The MAPE error value was 18%, which means that the model's predictions, on average, differ from the actual data by 18% of their value. The RMSE error is 3696.33, while the RMSPE error is 19.87%. The share of the MAE error in the forecasts was 5.44%, 4.77%, 4.18%, 3.66%, and 3.21%, respectively, while the share of the RMSE error in the forecasts was estimated at 6.6%, 5.79%, 5.07%, 4.44%, 3.89%, respectively.

Based on the analysis of forecast estimation using the analytical method, the most reliable seems to be the polynomial model for which the smallest errors were estimated: MAE 1713.47, MAPE 9.85%, RMSE 2160.92, and RMSPE 11.11%. For this model, the smallest standard error of the model wat the level of 2343.85.

2.5. Holt's model

The paper presents a prognostic analysis based on an analytical model. The following section presents the use of the Holt model to estimate forecasts for the following years, including the determination of errors (Zeliaś, Pawełek, Wanat, 2022). An Excel spreadsheet and the Solver add-in were used to find the optimal solution. Below are the formulas used in Holt's model:

a) *F* equation

$$F_t = \alpha \cdot y_t + (1 - \alpha) \cdot (F_{t-1} + S_{t-1}) \tag{15}$$

for t = 1:

$$F_t = y_1 \tag{16}$$

b) *S* equation

$$S_t = \beta \cdot (F_t - F_{t-1}) + (1 - \beta) \cdot S_{t-1} \tag{17}$$

for t = 1:

$$S_1 = y_2 - y_1 \tag{18}$$

c) Forecast for t > n:

$$y_t^* = F_n + (t - n) \cdot S_n \tag{19}$$

for 2 <= t < = n

$$y_t^* = F_{t-1} + S_{t-1} \tag{20}$$

Table 12 below shows the values of the *F* and *S* equations, expired forecasts, and individual error factor values for the initial value of the alpha and beta parameters equal to 0 (before running the solver).

Table 12.
Data values, F and S equations, expired forecasts and error factors for the Holt model

t	y_t	F_t	S_t	y_t *	MAE	MAPE	RMSE	RMSPE
1	2936.03	2936	686	-				
2	3621.60	3622	686	3622				
3	4284.07	4307	686	4307	23.09	0.01	533.29	0.00003
4	5087.22	4993	686	4993	94.48	0.02	8927.23	0.00034
5	6543.51	5678	686	5678	865.21	0.13	748581.42	0.01748
6	8508.58	6364	686	6364	2144.71	0.25	4599772.41	0.06354
7	10199.50	7049	686	7049	3150.05	0.31	9922815.00	0.09538
8	12752.83	7735	686	7735	5017.82	0.39	25178477.41	0.15482
9	16855.42	8421	686	8421	8434.84	0.50	71146458.35	0.25042
10	17028.46	9106	686	9106	7922.31	0.47	62762948.20	0.21645
11	19858.49	9792	686	9792	10066.76	0.51	101339717.30	0.25697
12	21978.98	10477	686	10477	11501.68	0.52	132288711.83	0.27385
13	22435.08	11163	686	11163	11272.22	0.50	127062876.10	0.25244
14	22537.50	11848	686	11848	10689.06	0.47	114256089.20	0.22494
15	22839.21	12534	686	12534	10305.21	0.45	106197270.70	0.20359
16	24965.59	13220	686	13220	11746.01	0.47	137968844.89	0.22136
17	27681.84	13905	686	13905	13776.70	0.50	189797325.12	0.24769
18	30858.20	14591	686	14591	16267.48	0.53	264631035.69	0.27791
19	37190.80	15276	686	15276	21914.51	0.59	480245923.86	0.34721
20	43789.70	15962	686	15962	27827.84	0.64	774388901.69	0.40385
Sum					173019.98	7.25	2602545209.67	3.51

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the analyses performed, the alpha and beta parameters' values and the forecasts that were obtained were determined for various errors. The results are presented in the table below: Table 13.

Table 13.
MAE, MAPE, RMSE, and RMSPE error values, alpha and beta parameters, and forecasts for the following years

	MAE	MAPE	RMSE	RMSPE
Error	1098.342	5.85%	1484.469	7.50%
alfa	0.905	0.925	0.825	0.747
beta	0.980	0.829	0.980	0.980
Forecast t = 21	50506.269	50256.513	50455.892	50299.248
Forecast t = 22	57307.055	56815.602	57352.659	57239.112
Forecast t = 23	64107.842	63374.691	64249.426	64178.975
Forecast t = 24	70908.628	69933.781	71146.193	71118.838
Forecast t = 25	77709.415	76492.870	78042.960	78058.701

Source: own study based on https://ec.europa.eu/eurostat/databrowser/view/nrg_ind_ured__custom_14213015/default/table

Based on the obtained results, comparing the MAE error and RMSE, a smaller value was obtained for the MAE error with forecasts for the following years: 50506.269, 57307.055, 64107.842, 70908.628, 77709.415, respectively. Looking at the MAPE and RMSPE percentage errors, a smaller value was determined for the MAPE error, for which the forecasts were set at the following levels: 50256.513, 56815.602, 63374.691, 69933.781, 76492.87. The share of the MAE error in the forecasts was, respectively, for the following years: 2.17%, 1.92%, 1.71%, 1.55%, 1.41%, while the share of the RMSE error in the forecasts was estimated at the following levels: 2.94%, 2.59%, 2.31%, 2.09%, 1.9%.

3. Summary

This article presents a detailed analysis and forecasting of gross electricity production in Poland in the context of implementing the assumptions of the Renewable Energy Directive. Various forecasting methods were used, such as linear, polynomial, power-law, exponential, and Holt models, to accurately assess the dynamics of changes in the renewable energy sector. The analysis results indicate significant differences in the effectiveness of individual forecasting methods. Comparison of forecast errors such as MAE, RMSE, MAPE, and RMSPE made it possible to assess the accuracy of each model and indicate the best approach depending on the time perspective.

The methods used provide valuable conclusions for planning the development of the renewable energy sector in Poland. Forecasts indicate a further increase in renewable energy production, which confirms progress in achieving EU goals. At the same time, the results emphasize the need to invest in energy infrastructure, further develop renewable energy, and integrate the power system.

The analyses can serve as the basis for making strategic decisions in the energy sector, supporting the transformation towards sustainable development and climate neutrality.

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