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APPLICATION OF THE METHOD OF MULTI-VALUED LOGIC TREES IN THE METHODOLOGY OF GEAR PUMP DESIGN

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Purpose: The aim of this article is to present a method of optimization of a discrete gear pump, using multi-valued logic trees (this is unique and original approach to this issue).
Design/methodology/approach: In this paper, we have presented the application of multi-valued decision logic in the design methodology of gear pumps. We are talking about the methodological process of „design - construction - optimization" beginning with the consideration of coupling relations between interacting assemblies, subassemblies and elements.
Findings: The developed methodology of multi-valued logic trees and parametric graphs in particular provides a new method for discrete optimization. The analytical description of the object speeds up finding the optimal solution and significantly reduces the cost of the optimization process.
Practical implications: The paper presents a methodology for designing gear pumps using multi-valued logic trees. The solutions obtained by means of the adopted methodology have been structurally supported by two patent applications and industrial implementation.
Originality/value: We have described the methodological cycle of application of multi-valued logic trees which resulted in development of a whole series of innovative new generation gear pumps. Finally, we introduced multi-valued weighting coefficients, which allowed us to develop innovative prototype pumps.
Keywords: design methodology, design method, graphs and trees, machine systems, design theory, multi-valued logic trees, gear pumps.
Category of the paper: Case study.

1. Introduction

Engineering practice requires a correct assessment of the mathematical model describing a given system by means of variables. Models describe a given system with different accuracy.

The analysis of design methodology, the structure of this process and the corresponding stages, show that methodological elaborations create the basis for the development of new design methods. Undoubtedly, the foundation for the creation of methods of design support has become heurestic procedures, which include all the rules and laws that help to solve the problem, but not always guarantee finding the best solution. The method of morphological analysis (Zwicky, 1969) and the method of solving invention tasks (Altszuller, 1972) should be mentioned here first of all. Morphological analysis is a creative problem-solving technique for systematically structuring and exploring all possible solutions to a multidimensional, complex problem and a powerful tool for generating creative ideas and designing new product and services. It is based on attribute analysis, generating alternatives for each attribute, creating new possibilities. Among other things, this work laid the foundation for algorithmic procedures based on logical inference.

Algorithmic design is based on determining the sequence of individual operations starting from the conceptualization stage ending with the creation of a task-based optimization criterion. This approach describes systems analysis. Rene Descartes' ideas in his Discourse on the Method (Descartes, 1637) greatly influenced design thinking: „divide every difficulty into as many parts as are possible and necessary for its adequate solution". A reference work that should also be cited is the book (Jones, 1992) describing a set of tools and a theoretical framework to support design. The representation of the design process by (Maher, 1996) cited by (Cross, 2008) was based on abductive thinking associated with production (synthesis), while induction and deduction are associated with analysis. It is clear that in many situations the design process will depend primarily on the designer.

W. Gasparski in (Gasparski, 1979) wrote that on the grounds of design methodology (methodology of solving practical problems) the object of scientific search for solutions is the art of practicality. Searching is understood here as solving problems arising from practical situations. In turn, the search is made in connection with the theory of decisions as „... praxeology as a theoretical discipline is, or becomes, an optimization theory of action ...", i.e. a discipline establishing optimization theories with an analytical basis in the form of decision theory. Therefore, the concept of design determined by the results of methodological studies so far was called praxeological-system concept.

The result of methodological observation of the ways of formulating hypothetical solutions-projects is the identification of two attitudes:

- the refining attitude,
- the idealizing attitude (the so-called projecting attitude),

which is schematically presented in Figure 1.

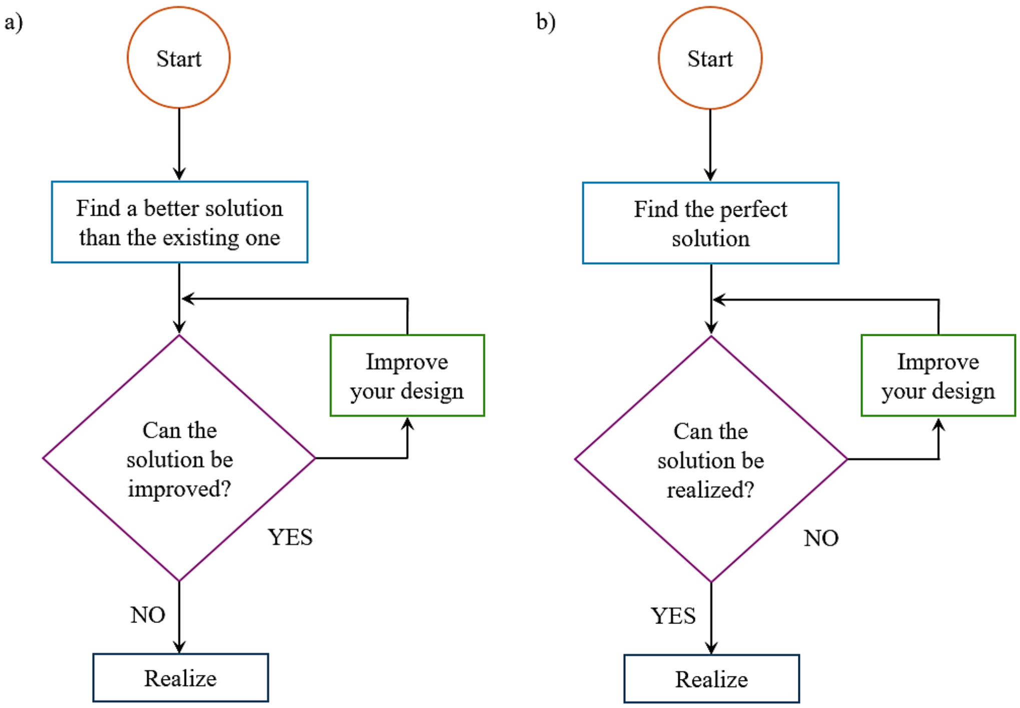


Figure 1. Two attitudes of formulating hypotheses to solve a design problem: a) the streamlining attitude, b) the designing attitude.

Source: Gasparski, 1979.

Analysis of the design process requires a precise definition of the essence of the process. The problem, „what is design?" is not a new one. The design attitude is characterized by the modern concept of design, based primarily on the formulation of a hypothesis of a solution that the designer thinks is ideal. A system is defined by a set of variables, a set of possible states corresponding to each variable, and by some way of describing the meaning of the variables and their states in terms of attribute valence. Systems whose variables are divided into input variables (we) and output variables (out) are called directed systems. The problem to be addressed can be roughly described as follows: given a data system, determine the best representation of that system by a directed structural system whose elements correspond to subsets of the data system variables.

Nowadays, the design method based on the development of technical diagrams is a widely used method. A paper (Daalhuizen, 2021) presents ideas for a new understanding of methods in design. The authors proposed Method Content Theory as a very powerful „indicator method" for education and practical applications along with the potential of useful insights in design methodology. A very important issue was addressed in (Ge, 2021), where the authors investigated emotions in design considering the preferences of experienced designers. They presented results showing how situated emotions mediate design work using a multi-level model analysis. This is extremely important primarily by considering the personality traits of the expert in risk assessment of innovative solutions in integrated decision support systems (Deptuła, Deptuła, 2017).

2. Literature review

Methodological thinking includes two aspects. The reflective aspect stems from a person's desire to understand and organize cognition from the perspective of its possible forms. The second aspect is a pragmatic orientation toward the practical solution of a particular problem. Thinking about method cannot be separated from thinking about logic. Virtually always the two concepts appear together in philosophy. The acceleration of the development of technology, taking place in the twentieth century, had a significant impact on the processes of preparation of practical actions taken by man. This acceleration resulted primarily from the complication of erected objects and their increasing scale. At the same time, a need arose to increase the efficiency of design, that is, to reduce its time, while achieving greater suitability of the designed object to the functional and economic requirements.

In other words, the need for project evaluation procedures has arisen. In recent years, the amount and degree of difficulty of design work have been growing faster than the capabilities of designers. This is especially true for machinery, equipment and production systems with complex applications, construction and operating principles. In this paper (Heiss , Kokshagina, 2021), the authors investigated the role of tactile co-design tools that enable interdisciplinary collaboration in complex healthcare environments. Specifically, they presented how the capabilities of tactile co-design tools can support equity in decision making, improve literacy in teams.

The need for the development and use of alternative design methods is caused not only by the increase in the requirements for machines and mechanical systems (interdisciplinarity, modularity, economy), but also by the weakness of the design process, which currently relies primarily on the experience, intuition of designers and often too much trust in computer programs and calculators, too little draws on the achievements of design methodology. Often, property modeling and optimization in mechanization and automation are not comprehensive.

In the design of machines and mechanisms, the object is a device composed of closely integrated parts that together realize the assumed (set) functions. An important feature of mechanical systems is the complex arrangement of geometric, technological and functional features. Small changes in the properties of the elements may cause large changes in the properties or manufacturing costs of the machine. Often, the designed device is an integral part of a larger system and must be properly matched. The task of the designer-constructor is to design a device performing strictly specified functions under many constraints.

In recent studies, diverse optimization techniques for gear pumps have been proposed. Zharkevich et al. (2023) presented a parametric optimization approach for gear pump casings using finite element analysis (FEA), achieving up to a 16% reduction in structure weight while maintaining structural integrity. Our study also applied FEA, yet we expanded upon this by optimizing stress distribution to improve durability and resilience under dynamic loads. Similarly, Guerra et al. (2021) introduced a methodology for designing multistage gear pumps intended for dry sump systems, which included detailed structural analysis of the drive shaft and internal clearances. While we also performed structural analysis, our research incorporated novel gear profiles and clearance compensation mechanisms to achieve enhanced stability and performance efficiency. Dhote et al. (2022) explored the use of the Taguchi method to optimize parameters such as material hardness, outlet pressure, and rotational speed in order to minimize wear. Although our study also analyzed materials, we emphasized reducing cavitation and improving hydromechanical parameters through advanced structural optimization. Robison (2021) developed a multi-objective optimization technique tailored to gerotor geometry, with goals to reduce flow ripple and leakage while enhancing efficiency. Inspired by this approach, we applied a similar optimization framework within the context of gear pumps, ultimately producing prototype pumps with substantially improved flow stability. Another study by Nguyen et al. (2022) utilized CFD simulations and response surface methodology to optimize centrifugal pumps for slurry transport, successfully enhancing efficiency and reducing erosion. In our research, we also leveraged advanced CFD simulations, focusing specifically on optimizing gear pump designs to achieve significant noise and pulsation reductions.

Garcia and Patel (2022) conducted research on the use of magnetorheological fluids for dynamic sealing in gear pumps, which led to enhanced seal performance. Although we focused on conventional sealing methods, similar optimization principles were applied in our study to improve the overall seal tightness and efficiency. An analysis by Cinar et al. (2016) used FEA to investigate stress distribution within the pump casing, identifying high-risk zones for damage under various load conditions. In a similar vein, we applied FEA but implemented additional design modifications to reduce stress concentration and increase the durability of the pump. Smith and Jensen (2023) demonstrated that modifications to tooth profiles could lower noise levels in hydraulic pumps by up to 15%. Expanding on this, we incorporated clearance compensators, achieving even greater acoustic improvements—a crucial factor in noise reduction for industrial applications. In examining high-pressure gear pumps, Kim and Wei (2023) investigated strategies to reduce cavitation. Following their insights, we developed additional design solutions that both minimized cavitation and improved hydraulic efficiency.

Ivanović et al. (2021) focused on optimizing parameters of gerotor pumps to reduce wear and enhance efficiency. Building on this approach, our research aimed to achieve similar improvements in gear pumps through structural optimization and advanced materials. Further, Zharkevich et al. (2023) once again applied FEA to gear pump casings, achieving a 16% weight reduction while ensuring structural strength. In our research, we not only pursued weight reduction but also aimed to enhance efficiency and durability, thus broadening the impact of their findings.

Orlandi et al. (2023) developed a methodology for CFD simulation of gear pumps, accurately modeling the contact between gear teeth under actual operating conditions. Our study also employed CFD simulations, but with a distinct focus on flow optimization and reducing performance pulsations. Pourmostaghimi et al. (2023) used evolutionary optimization techniques for reverse-engineering helical gears, successfully determining critical design parameters. Although their research concentrated on gear shape reconstruction, we adapted similar techniques to enhance key performance parameters in gear pumps. In a related area, Peng et al. (2023) studied the influence of radial and end clearances on the efficiency of high-speed gear pumps, identifying optimal values to reduce internal leakage and frictional losses. In our study, clearance optimization was similarly addressed, but with a broader emphasis on durability and noise reduction. Cao et al. (2023) applied CFD alongside optimization methodologies to analyze internal flow characteristics in a guide vane centrifugal pump. While our focus remained on gear pumps, we applied a similar approach to flow modeling, leading to substantial performance gains. Wei Li et al. (2024) proposed a combined approach using response surface optimization and CFD to improve energy characteristics and flow states during pump startup, thereby enhancing transient efficiency. Although our scope was different, we also focused on optimizing dynamic parameters in gear pumps.

Finally, Dixit et al. (2023) introduced a multi-objective optimization method for designing spur gears, accounting for multiple failure modes. Our study builds upon this framework, particularly in optimizing structural strength and efficiency in hydraulic gear pumps.

Our study is unique in that it introduces multi-valued logic trees as a method for optimizing gear pumps. The “design - construction - optimization” process has been refined through the use of coupling relations between interacting assemblies, subassemblies, and components. By using multi-valued logic trees and parametric graphs, we were able to rapidly and effectively identify optimal solutions, accelerating the design process and significantly reducing costs. The developed methodology has resulted in two patent applications and has been implemented in industrial settings, emphasizing its practical significance.

The introduction of multi-valued weighting coefficients within the optimization process is another novel element, allowing us to develop innovative prototype gear pumps for the next generation. The contribution of our research to the scientific field includes expanding machine design theory and introducing new tools for analyzing and optimizing complex mechanical systems.

2.1. Graph-based decision - making models

Design decision support tools include decision trees, dendrites, tree classifiers, and graphs, among others. These tools are classified as so-called graphical decision support and decision making methods (Jones et al., 2001).

Often, a set of decisions (and the relationships between them) are written graphically from a mathematical model, which are the main implementation plane of the decision making process that a decision maker can use to solve any problem (Cheng et al., 2018). This usually refers to the stage in the early stages of the design process where schematics, sketches, and prototypes exist. In a paper (Segers et al., 2005), the authors effect of offering feedback to annotations by, among other things, presenting word graphs that contain the architect's annotations and semantic associations based on those words. The authors showed that the feedback provided by word graphs can stimulate design. The application of generating parametric trees for a given graph playing out parametrically for automatic transmissions is presented in (Deptula, Kurmayev, 2021).

For decision support in optimization of mechanical systems, modeling of the overall process is required. The most commonly used graph models in the analysis of mechanical systems are graphs and decision trees (Cai, 2010). There are a number of publications in the literature on the applications of decision trees in decision making systems in the methodological design and construction process. This is mainly related to meta-design theory, considering parametric modeling using graph methods, among others. In a paper (Harding, Shepherd, 2017), the authors proposed a new approach called Meta-Parametric Design, combining graph-based parametric modeling with genetic programming. The advantages of this approach are demonstrated through the example of two real case study projects, which extend the exploration of the design while maintaining the benefits of graph representation.

In particular, multi-valued logical decision trees can be distinguished in design (Deptuła, 2020; Deptuła et al., 2019).

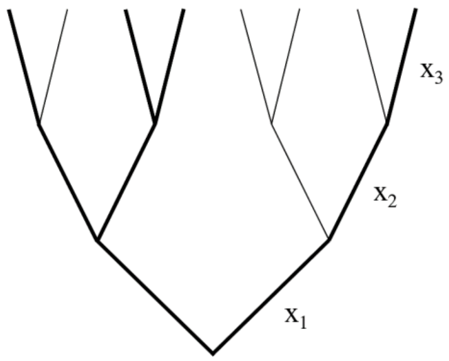
3. Multi-valued logic trees as graph models in machine systems analysis

3.1. The development of logical binary trees - Boolean algebra

The method of multi-valued logic trees is effectively used in discrete optimization of machine systems. When each design and/or performance parameter taking numerical values from a certain interval is labeled with a fixed binary or multi-valued logical variable, a discretization of such numerical intervals can be performed. The set of all numerical combinations forms a variant tree with a number of floors equal to the number of design and/or performance parameters, since in the case of a traditional logic tree, there can be only one logical variable on a single floor. The number of sub-branches of a given interval represents the number of branches in a single branch bundle, and the number of all paths from the bottom to the top of the logic tree corresponds exactly to the number of all combinations of discrete values of the considered intervals.

The method of multi-valued logic trees results from the development of logical decision trees and Boolean algebra (Béziau, 2012). Based on Boolean algebra, two important branches of mathematics like multiplicity theory and classical logic were defined. In this algebra, basic symbols, axioms and the set of theorems derived from them are defined. The set of axioms can be chosen arbitrarily. The use of multiplicity theory and Boolean algebra allows, among other things, to make descriptions of the design process based on the analysis and synthesis model. For example, in the work (Darke, 1979) the author proposed to develop the paradigm - conjecture analysis o in order to create a model of the design process consisting of generator-concept - analysis. The methodology of multi-valued logic trees also allows a similar approach.

Figure 2 shows a logic tree that encodes a fixed Boolean function of three variables.



$$f(x_1, x_2, x_3) = x_1 x_2 x_3 + \overline{x_1} x_2 x_3 + \overline{x_1} x_2 \overline{x_3} + \overline{x_1 x_2 x_3}.$$

Figure 2. Boolean function of three variables encoded on a logic tree.

Source: based on own research analysis.

In the Quine - McCluskey algorithm, by simplifying the Boolean functions written in KAPN, the truncated alternative normal form (SAPN) and finally the minimum alternative normal form (MAPN) are obtained (Figure 3).

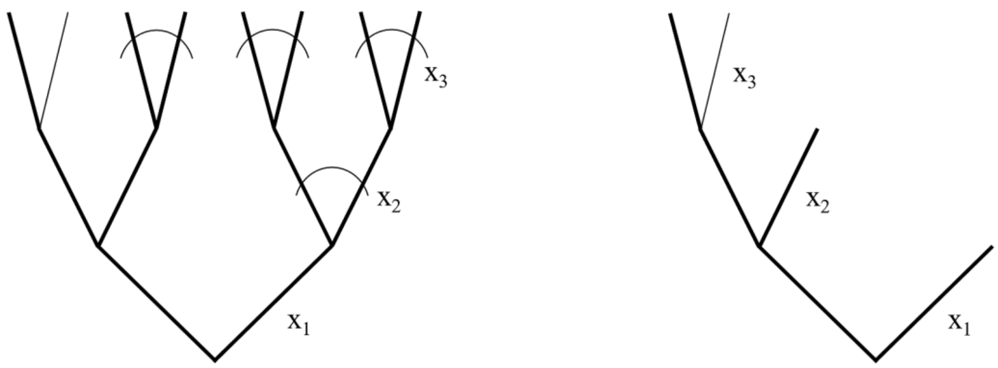


Figure 3. Logic tree and simplified logic tree.

Source: based on own research analysis.

The minimized form of the output function (with the minimum number of literals) is then obtained.

3.2. Quine - McCluskey algorithm for minimization of partial multi-valued logical functions

In the minimization considering multi-valued complete logic systems, the rank of importance of fixed variables, detailed analysis of realizable sub-solutions, etc. are determined. In the case of multi-valued Boolean functions, as in Boolean functions, the notions of incomplete gluing and elementary absorption, which are applied to the APN of a given Boolean function, play a fundamental role in the search for prime implicants.

The Quine - McCluskey algorithm makes it possible to find all prime implicants of a given logic function that is there is a shortened alternative normal form SAPN (Bhandaria et al., 2018). The terms of incomplete gluing and elementary absorption have the main role in the search of prime implicants and are used for the APN of a given logic function (Partyka, 1983, 1999). The following transformation is called the consensus operation:

$$Aj_o(x_r)+\ldots+Aj_{m_r-1}(x_r) = A \tag{1}$$

where:

$r = 1,\ \ldots\ ,\ n$

A – a partial elementary product, the literals of which possess variables belonging to the set: $\{x_1,\ldots,\ x_{r-i},\ x_{r+i},\ldots,\ x_n\}$.

The following transformation is called the operation of reduction:

$$Aj_u(x_r) + A = A \tag{2}$$

where:

$0 \le u \le m_r - 1;\ 1 \le r \le n,$

A – a partial elementary product, the literals of which possess variables belonging to the set: $\{x_1,\ldots,\ x_{r-1}, x_{r+1},\ldots,\ x_n\}$. If the above equation takes place, then A absorbs $Aj_u(x_r)$. In the case of multi-valued weighting factors, we get:

$$\begin{aligned} &w_o Aj_o(x_r)+\ldots+w_{m_r-1}Aj_{m_r-1}(x_r) = \\ &= \left(\min\{w_o,\ldots,w_{m_r-1}\}\right)\cdot A + \sum_{s=i_o,\ldots,i_{m_r-2}} w_s\cdot A\cdot j_s(x_r) \end{aligned} \tag{3}$$

where w_i - polyvalent weighting factor.

For example using the formula:

$$\begin{gathered} Aj_0(x_r)+\cdots+Aj_{m-1}(x_r) = A, \\ Aj_u(x_r) + A = A \end{gathered} \tag{4}$$

where:

$$A = A(x_1,\ \ldots,\ x_{r-1},\ x_{r+1},\ \ldots,\ x_n), \tag{5}$$

$$j_u(x_r) = \begin{cases} m-1, u = x_r \\ \qquad\qquad\qquad 0 \le u \le m-1 \\ 0, u \ne x_r \end{cases} \tag{6}$$

successive stages of the multi-valued logic function minimization: 020, 101, 200, 021, 111, 201, 210, 022, 121, 202, 211, 212, 221 can be presented in the following way (Table 1).

Table 1.
NAPN and MAPN of a given logical function

	020	200	101	021	201	210	111	022	121	202	211	212	221
02-	*			*				*					
20-		*			*					*			
1-1			*				*		*				
21-						*					*	*	
-21				*					*				*
2-1					*						*		*

Source: Partyka, 1983, 1999.

3.2.1. Consideration of multi-valued weighting factors for the Quine - McCluskey algorithm

As in the case of minimization of multi-valued Boolean functions without weighting coefficients, in the algorithm elementary products are written as numbers in appropriate positional systems. The author's algorithm for minimizing multi-valued Boolean functions with weighted coefficients is presented in the book (Deptuła, 2020). For the gluing operation of individual partial multi-valued logical functions with weighted coefficients, definitions of „pure" and „impure" gluing are introduced, where the gluing operation for canonical multi-valued elementary products is performed with respect to the weighted coefficient with the smallest value, i.e. $min\{w_1, \ldots, w_n\}$.

An impure gluing operation is a transformation:

$$w_o Aj_o(x_r)+\ldots+w_{m_r-1}Aj_{m_r-1}(x_r) = \\ = \left(\min\{w_o,\ldots,w_{m_r-1}\}\right)\cdot A + \sum_{s=i_o,\ldots,i_{m_r-2}} w_s \cdot A \cdot j_s(x_r) \qquad (7)$$

where:

$\mathrm{r} = 1, \ldots, n;\ w_s > min\{w_0, \ldots, w_{m_r-1}\}$,

A – elementary partial product whose variables of particular literals belong to the set $\{x_1,\ldots,x_{r-1},x_{r+i},\ldots,x_n\}$. In n variables $(m_1,\ldots,m_n)$ - value weighting factor before the partial canonical product takes values in the range $\langle w_1, \ldots, w_n\rangle$, whereby $w_j = w_{j-1} + w_{j-2} + \ldots + w_1$ where $j = 2,\ldots,n$.

4. Gear pump design methodology using multi-valued logic trees

4.1. Gear pumps

Gear pump designs have been developed for over 400 years. The first descriptions of gear pump design appeared in works: Hilaria mathematica (1624), Recreation mathematique, composee des plusieurs problemes plai sants et facetieux (1626), whose authorship is attributed to Jean Leurechon (Beck, 1901).

The creator of the first gear unit was Johannes Kepler, who patented his solution in 1604. The initial application of the patented solution at that time was the pumping of water in dewatered mines (Abel, 1971; Stryczek, 2007). Valveless and self-priming gear construction resistant to numerous impurities present in water removed from shaft sumps very quickly displaced piston pumps used at that time and troublesome in operation. The author of the solution saw a wider application of his invention. He did not exclude the use of his idea for removing bilge water on ships, forcing the flow in the installation of park and garden fountains, but also for pumping air in the construction of blowers and exhaustors. Thus, as early as the 17th century, great importance was placed on maintaining proper dimensional tolerances of the mating displacement elements with the body.

Improving internal tightness is a direction that is associated with minimizing energy losses, increasing transmitted power, and increasing generator energy efficiency (Amman, 1926; Arai, 1968; Tilley, Burrows, 1995). The total efficiency of gear pumps produced today is about 80-90% (for nominal pressures up to 28 MPa). Such a large range is mainly associated with the adopted manufacturing tolerances. Due to the considerable difficulty of correlating in a simple way the effect of tolerances of individual elements on the energy efficiency of the produced pump.

Gear pumps are the most common power generators used in hydraulic power transmission systems, whose share is estimated at more than half of all pumps manufactured. The most important advantages of gear pumps are simple and compact design, reliability of operation, resistance to contamination of the working medium and high efficiency coefficient at the same time with small dimensions compared to other pumps. Figure 4 shows the division of gear pumps by application (Osiński, 2013).

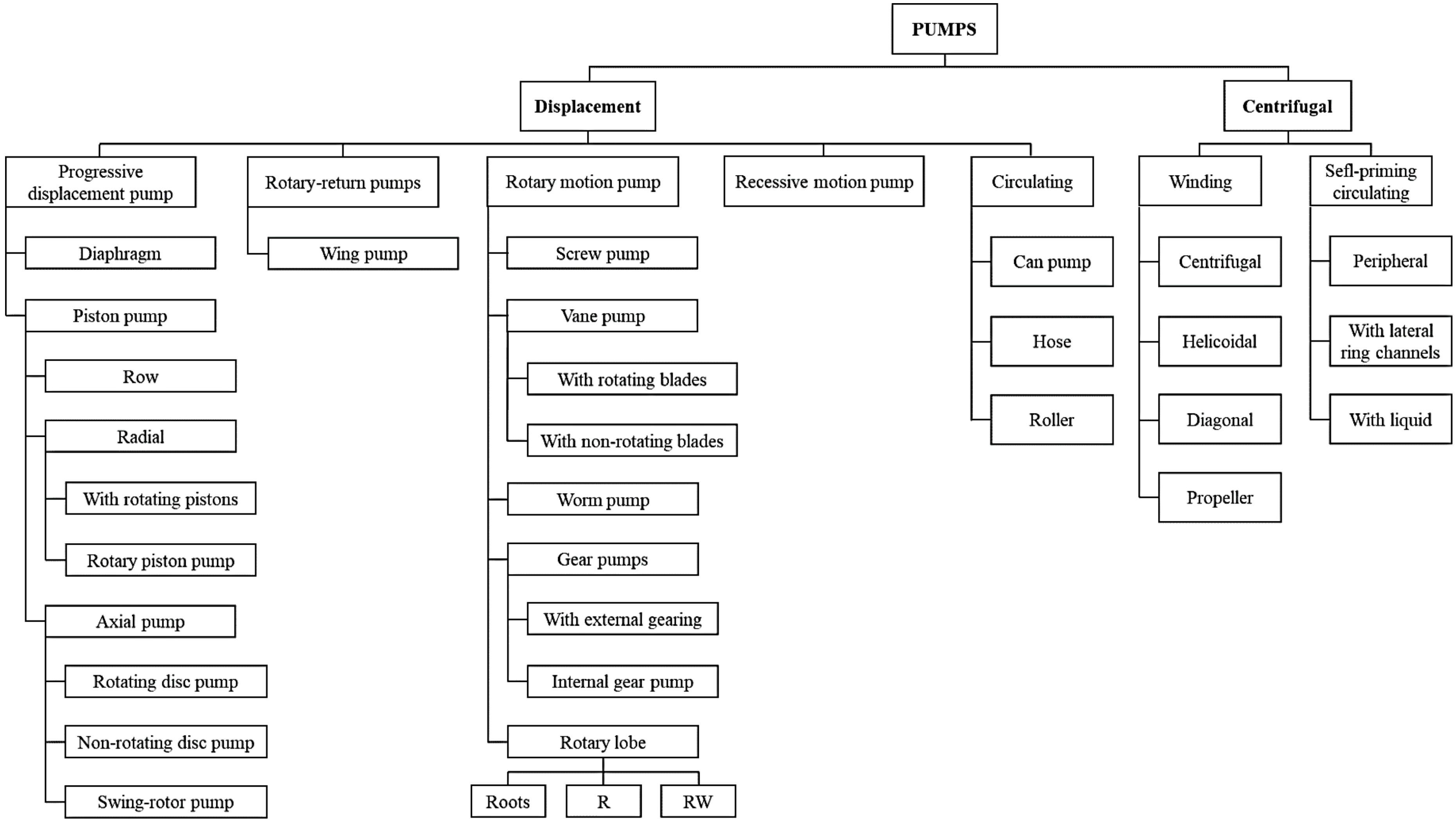


Figure 4. Classification of centrifugal and positive displacement pumps.
Source: Osiński, 2013.

Despite the existence of numerous patents, literature and a huge number of currently manufactured gear units, technical methods to ensure optimal internal tightness, maximum operating pressures and minimum performance pulsation and noise emission have not yet been exhausted. In particular, the paper (Osiński, 2013) presents possibilities and methods to reduce performance pulsation and to propose new design solutions to increase operating pressures while ensuring high internal tightness. These tasks required solving a number of technological, design and construction problems.

For a pump to be widely used in hydraulic power transmission systems, it must meet, among other requirements:

- high operating pressures should be produced with the highest possible efficiency,
- adequate and least fluctuating capacity should be ensured over the entire operating pressure range.

Realization of these requirements is connected with ensuring the highest possible internal tightness, which also influences the value of the overall efficiency of the gear pump. The efficiency of the pump is closely related to the energy losses occurring during the operation of the unit. The efficiency of positive displacement pumps is affected by many factors of design, technology and operating conditions. The most important of these are:

- operating medium and operating conditions-hence the research methodology presented in (Osiński et al., 2013),
- the toothing parameters and the design of the sealed space relief-therefore the research on the optimization of the polyvolute outline presented, among others, in (Deptuła, Osiński, 2017),
- the quality of the mating surfaces, machining, and tolerances of the components-hence the research methodology undertaken in (Deptuła et al., 2017),
- the design of the new units involved several stages of retrofitting and testing of prototype pumps.

5. Application of multi-valued logic trees in the optimization of a discrete undercut gear pump

The paper (Osiński et al., 2013) describes the study of a prototype gear pump manufactured at Wytwórnia Pomp Hydraulicznych Sp. z o.o. in Wrocław. The innovation of the prototype unit consisted in modifying the profile of the evolute in its lower part by so-called undercutting of the tooth foot (Figure 5).

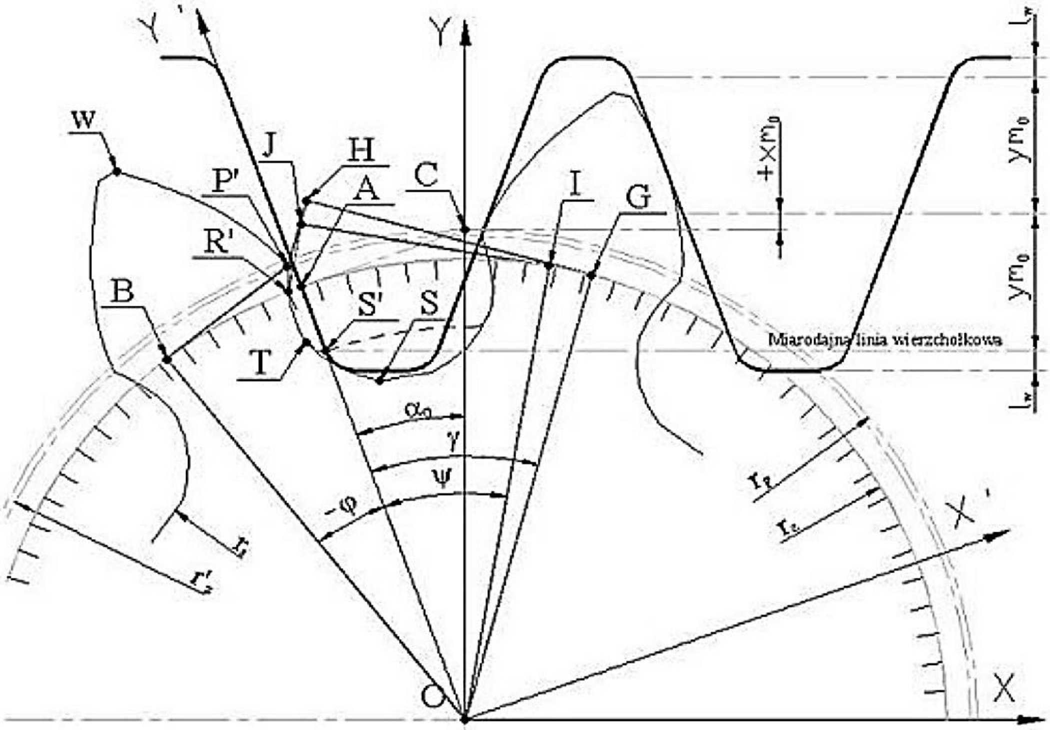


Figure 5. Undercutting the foot with a trapezoidal toothed tool.

Source: Kollek, Osiński, 2009.

In the model of the involute tooth outline (Figure 5), it was assumed that as a result of rounding or chamfering the cutting edge, the authoritative vertex line would be shifted in the direction of the tool foot radius by the value of the vertex clearance l_w.

The application of multi-valued logic trees in the optimization of a discrete gear pump presented in (Osiński et al., 2013) presents a new approach to the problem, as it was previously calculated by other methods (Kollek, 1996). In order to determine the optimal operating conditions and operating medium parameters, it was necessary to consider compartmental coding and the relationship between gear pump design parameters and operating parameters in a hierarchical approach.

The total efficiency can be expressed as follows:

$$\eta_c = \frac{1 - c_\mu \dfrac{p}{2\pi\mu \cdot n} - c_r \dfrac{1}{n}\sqrt{\dfrac{2p}{\rho}}\sqrt[3]{q^{-1}}}{1 + c_v 2\pi \dfrac{\mu \cdot n}{p} + c_\rho \dfrac{\rho \cdot n^2}{2p}\sqrt[3]{q^2} + c_p}. \tag{8}$$

In the search for the optimal value of the functions: $\eta_V, \eta_{hm}, \eta_c$ the corresponding arithmetic values of the ranges of change of the parameters under study were taken, which were encoded with logical decision variables in logical decision trees. If in the gear pump, with undercut tooth foot, all paths of multi-valued logical trees mean the set of all theoretical variants of the process of optimization of the corresponding efficiency η_c, η_{hm} and η_V, then only the true variants should be extracted (paths in bold - depicted graphically) (Figure 6).

Complex multi-valued logical functions by swapping the floors of the logic tree allow to determine the rank of importance of logical variables from the most important (at the root) to the least important (at the top), as there is a generalization of the bivalued quality indicator to multi-valued: (C_k - $k_i m_i$) + (k_i+K_i), where C_k - the number of branches of *k*-th floor, k_i - the multiplicity of simplification on *k*-th floor of the m_i - value variable, K_i - the number of branches of (*k-1*) - ego floor, from which the non-simplifying branches of *k*-th floor were formed. In this way, the minimum complex alternative normal form can be obtained. MZAPN of a given logical function, which has no isolated branches on the decision tree, and at the same time has a minimum number of true (realizable) branches, which in particular can be considered as elementary design guidelines (Figure 6). All transformations are described by the so-called Quine - McCluskey algorithm of minimizing individual partial multi-valued logical functions.

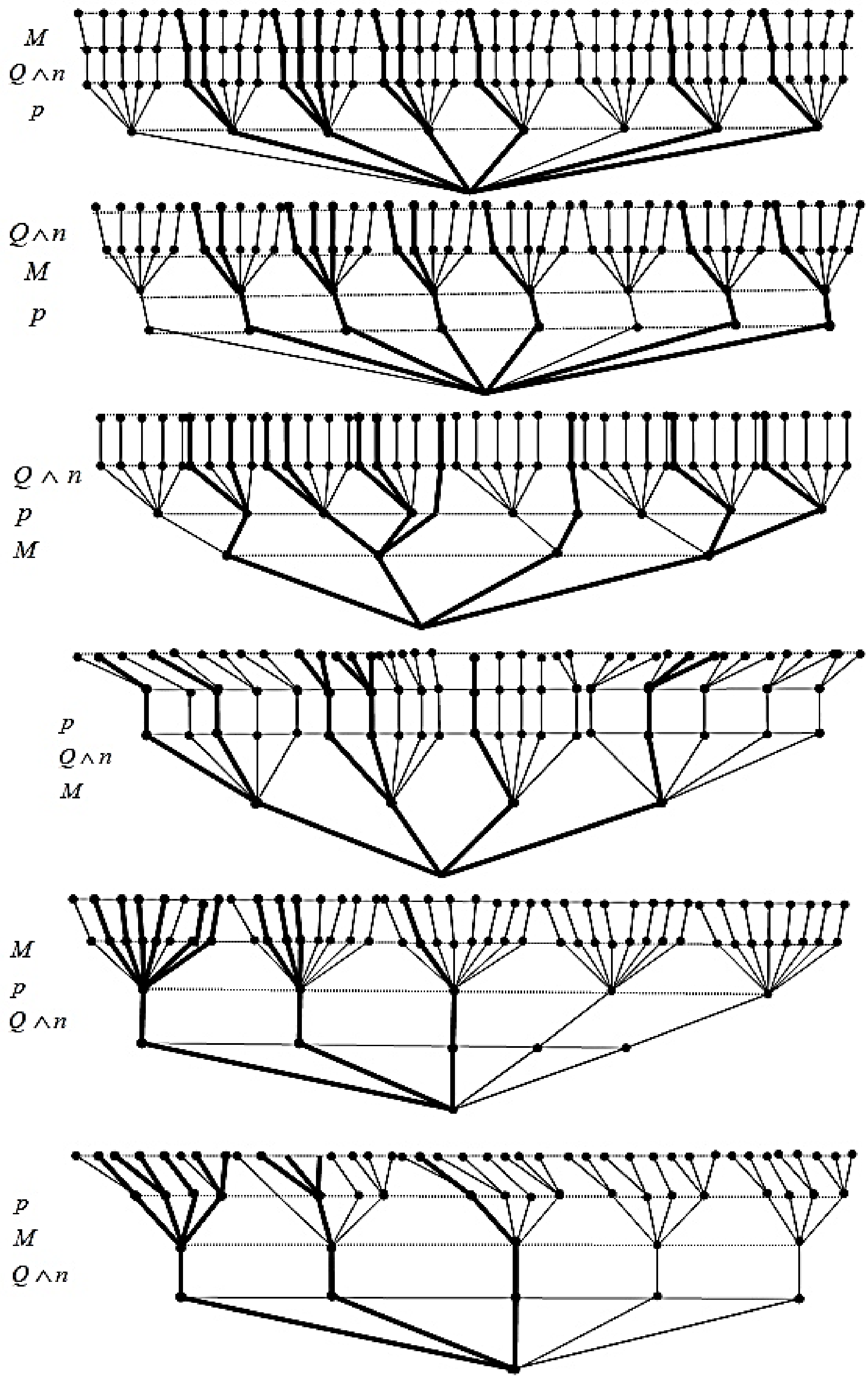


Figure 6. Multi-valued logic trees for hydraulic-mechanical efficiency η_{hm}.

Source: based on own research analysis.

If the number of realizable variants for the corresponding efficiencies with interchangeable floors with logical variables assigned to the design parameters *Q*, *n*, *M*, and *p* is calculated for the logic trees, only the logic trees with the least number of true branches describe the importance rank of the parameters, from the most important at the bottom to the least important at the top. Figure 7 shows the optimal multi-valued logic tree for volumetric efficiency η_V.

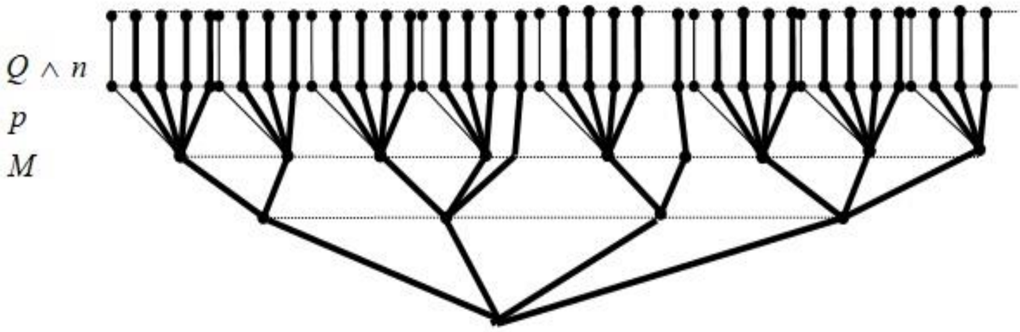


Figure 7. Optimal multi-valued logic tree for volumetric efficiency η_V.
Source: based on own research analysis.

It is shown in (Osiński et al., 2013) that for total efficiency there are two optimal multi-valued logic trees. The most important parameters are *n* and Q_{rz} - treated as one surrogate variable and the moment *M*. For mechanical and volumetric efficiency there are single optimal multi-valued logic trees. For mechanical efficiency, the most important parameters are n i Q_{rz} and for volumetric efficiency η_V, moment *M*.

In another source of knowledge (Deptuła et al., 2018), the presented analysis of the validity of the structural and operational parameters of the gear pump was supported by the results of alternative verification methods:

- analysis of additive and multiplicative multiple regression models,
- assessment of the importance rank of operational parameters based on the results of the Fisher-Snedecor test and the values of standardized BETA coefficients (Łuszczyna, 2011).

5.1. Optimization of a polyvent tooth outline

The phenomenon of seizing of the working fluid in the recesses of mating gear pairs occurs in pumps with external gearing when the number of adhesion is greater than unity. The optimized polyvolute outline should provide small changes in the dynamic force in the mesh as a result of the applied modifications and corrections. In addition, a smoother course of the dynamic force in the gearing can be influenced by an oblique tooth notch. When considering a polyvolute consisting of two normal involutes, the following cases are distinguished (Figure 8):

- the upper turret support angle has a larger value than the lower turret support angle $\alpha_0^g > \alpha_0^d$, such a solution has the advantage of relieving the confined space and causes a mitigation of the dynamic force changes in the meshing (change of stiffness of the mating pairs of teeth),
- the top turret lip angle has a smaller value than the bottom turret lip angle $\alpha_0^g > \alpha_0^d$, this solution results in worsening of the cap space relief and an increase in the dynamic force amplitude.

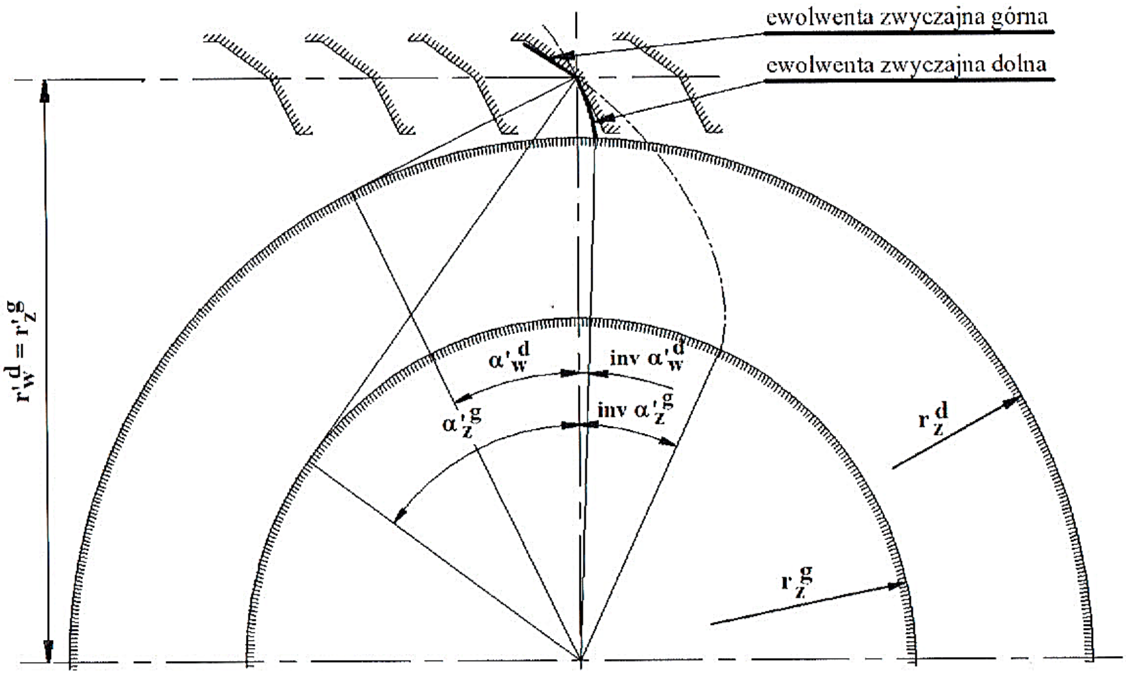


Figure 8. Two-evolution outline composed of ordinary polyevolutions.

Source: Osiński, 2017.

The optimization process was carried out taking into account five basic criteria: technological feasibility of the tool, obtaining a minimum compression ratio, occurrence of small changes of dynamic forces in the mesh, obtaining a minimum efficiency pulsation ratio and ensuring high energy efficiency.

Outline optimization using multi-valued logic trees required a two-step analysis (Deptuła, Osiński, 2017). For this purpose:

1. A computer model program was developed for the hydraulic parameters of the gear pump and to calculate the minimum, maximum, and instantaneous capacity and to generate the polyvolute outline.

2. A two-step approach was used in solving the problem using multi-valued decision trees:
   - application of multi-valued logic trees with weighted coefficients to determine the relationship between the $\alpha_0^1, \alpha_0^2, \alpha_0^3$ evanescent support angles in the polyvolute outline (stage I),
   - verification of criterion conditions and selection of variants of polyvolute outlines,
   - application of multi-valued logic trees to select the optimal variant of polyvolute outline (stage II).

For the study of multi-valued logic trees, a method taking into account weighting factors was additionally developed. In order to take into account the weight coefficients, the Quine-McCluskey algorithm of minimization of multi-valued functions was modified by introducing additional operations, such as definitions of impure gluing.

The impure gluing operation for multi-valued canonical elementary products is performed with respect to the weighting factor with the smallest value, i.e. $min\{w_1, \ldots, w_n\}$.

An impure gluing operation is a transformation of:

$$\begin{aligned} & w_o A j_o(x_r) + \ldots + w_{m_r-1} A j_{m_r-1}(x_r) = \\ & = \left(\min\{w_o, \ldots, w_{m_r-1}\}\right) \cdot A + \sum_{s=i_o,\ldots,i_{m_r-2}} w_s \cdot A \cdot j_s(x_r). \end{aligned} \tag{9}$$

In partial data of multi-valued logical functions $f_i(x_1, \ldots, x_n)$ n variables $(m_1, \ldots, m_n)$ - values, weighting factors had to be included in the gluing and pseudo-gluing operations $(w_n,\ w_{n-1},\ w_{n-2},\ \ldots,\ w_1)$, assigned to the corresponding multi-valued logical products.

A schematic of the polyvolute outline optimization algorithm is shown in Figure 9.

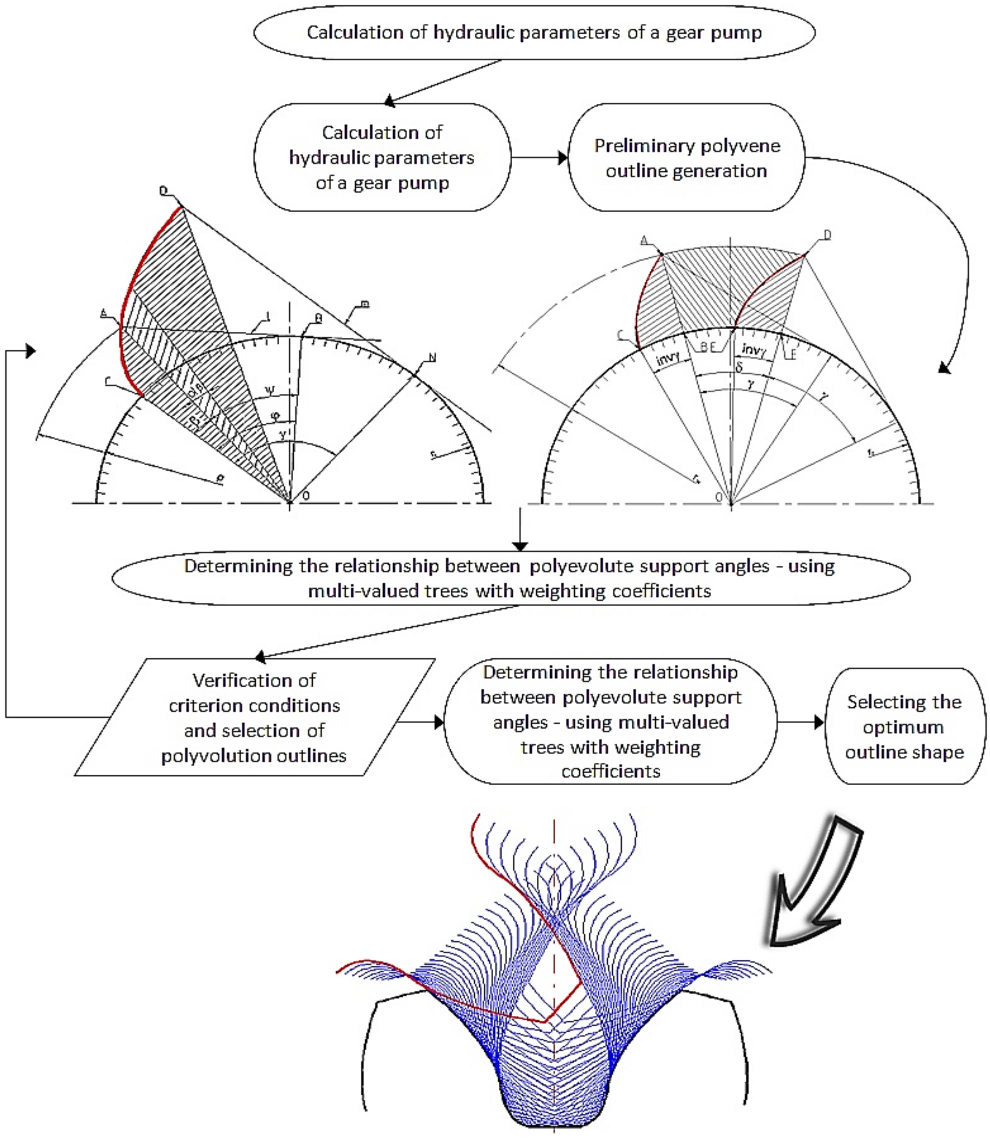


Figure 9. Schematic of the polyvolution outline optimization algorithm.

Source: Deptuła, Osiński, 2017.

In the first step, multi-valued logic trees with weight coefficients were used. 20 realizable solutions were obtained. Figure 10 shows an example of a multi-valued logic tree with weighting coefficients with stacked parameters α_0^1,α_0^2, α_0^3.

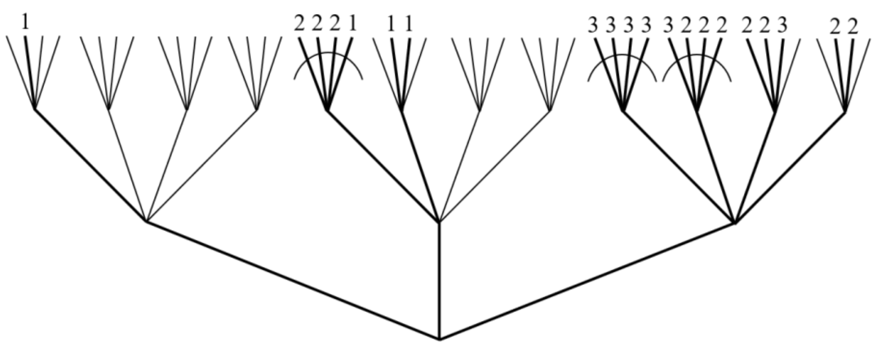


Figure 10. Multi-valued parameter logic tree: α_0^2, α_0^1,α_0^3. Minimization of solutions.

Source: based on own research analysis.

We then proceeded to determine the final shape of the polyvore outline. There were 216 theoretical solutions and $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$ multi-valued logic trees were obtained. The optimization process of the polyvolute outline indicated a solution of type *X1* (Figure 11) which consists of two ordinary evolutes located in the upper and middle parts and an elongated evolute causing undercutting of the tooth foot.

In addition, the optimization determined the correlations between the support angles of the usual evolutions, i.e. $\alpha_0^g > \alpha_0^d$ and $\alpha_0^d = 20°$. Based on the calculations and technological capabilities of Hydrotor S.A. (the entity implementing the design), a whole series of gear pumps with a three-rotor outline was designed. Shown in Figure 12 (Deptuła, Osiński, 2017) the model wheels were equivalent to the wheels designed for the pump of the second group with unit capacity $q = 8$ cm^3/rev.

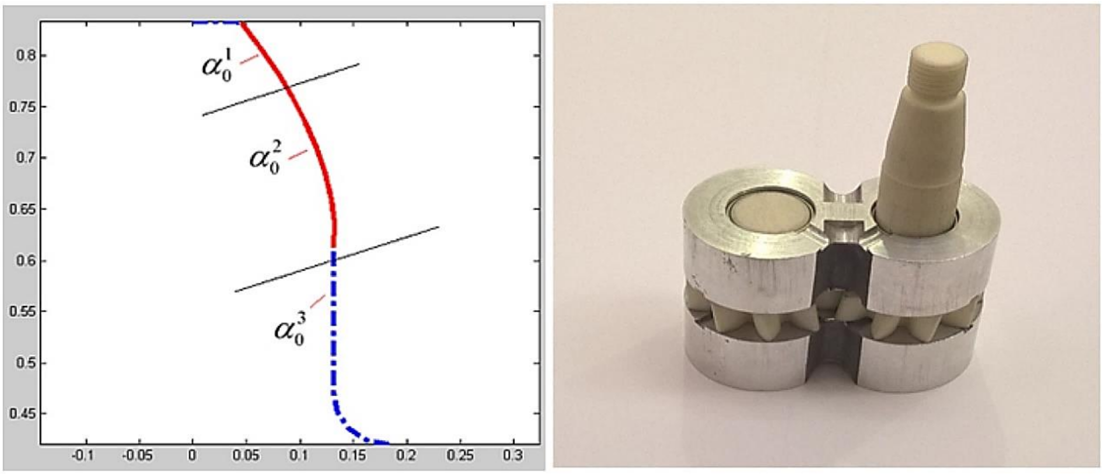


Figure 11. Polyvolute tooth outline consisting of three evolutes α_0^1, α_0^2, α_0^3 and three evolute wheels made with 3D printing technology.

Source: based on own experimental experience.

For such assumptions, the wheels should be corrected and the value of the correction factor $x < x_{min} = x_g$, this relationship implies the number of teeth $z_1 = z_2 < z_{min} = z_g$. The literature for strength reasons defines the maximum permissible value of correction factors according to the relation:

$$x_{max} = x_g' = y\frac{z_g' - z}{z_g}. \qquad (10)$$

Figure 12 shows an example of a dentition outline.

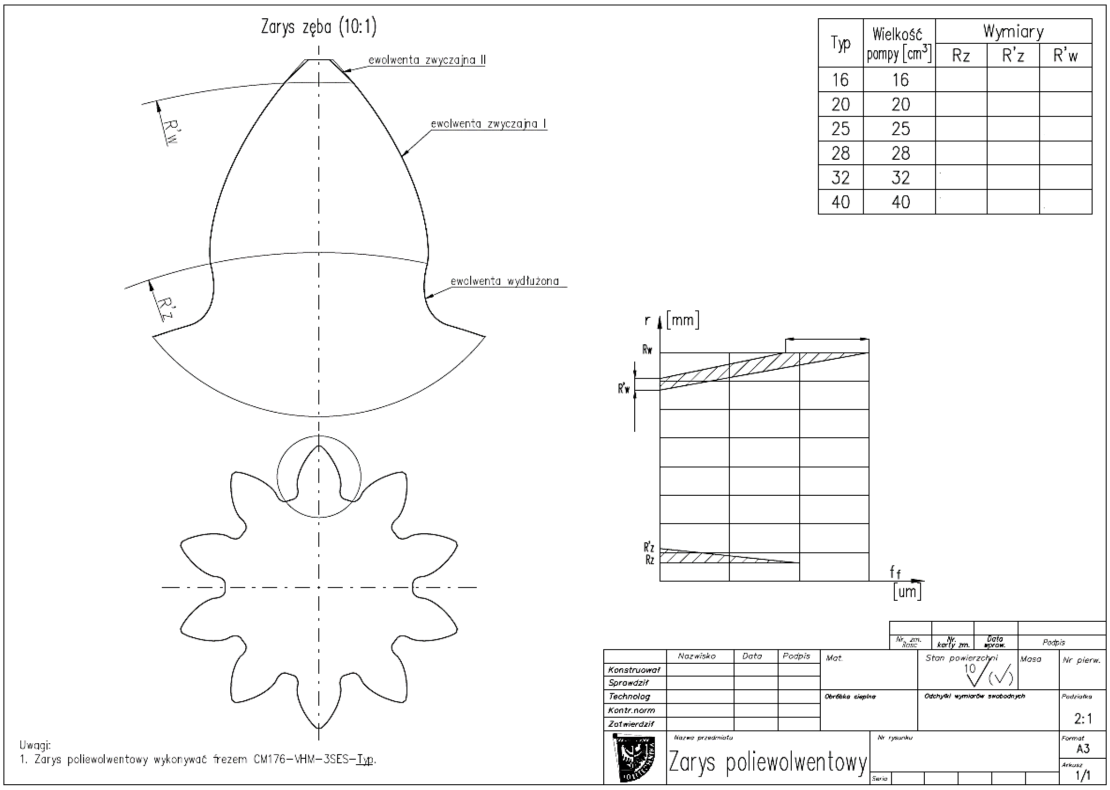


Figure 12. Structural drawing of an *X1*-type three-rotor outline.

Source: Deptuła, Osiński, 2017.

The manufacture of gear pumps with reduced acoustic emissions has required an increase in product accuracy. Attention is primarily given to increasing performance while increasing manufacturing accuracy. In order to achieve sustained accuracy at the micrometer level, a whole range of errors, i.e. kinematic errors, geometric errors induced by the cutting forces and the adopted machining parameters, must be continuously checked and compensated. Using multi-valued logic trees at a further stage in the work (Deptuła et al., 2017; 2020), optimization of the machining technology of the elements affecting the overall efficiency of the newly designed units was performed. Performed analysis of dimensional and shape tolerances in the end allowed to select control dimensions: critical, important and unimportant.

5.2. Optimization of processing technology for components affecting overall efficiency of newly designed units

The purpose of carrying out the identification of the influence of the technology of making polyvolute pump structures was to determine the sensitive control dimensions (values/tolerances) of the tested pumps for teeth made in the technology of chipping and grinding.

Model and prototype pump designs of the types *2PWR-SE*, *3PWR-SE*, *1PWR-SE* and *2PW-SEW* were the objects of the study. The paper (Deptuła et al., 2017) presents the results for the model pump of the 3PWR-SE series, while the papers (Deptuła et al., 2018a; 2019; 2018b) present the results for the other pump types.

In order to determine the influence of manufacturing technology on the level of noise emitted to the environment, it was decided to make gears in the ground chip technology

The computational methodology consisted first of determining the mutual effect of the given control measurements on the total efficiencies at given speeds $n = 500$, 800, 1000, 1500 and 2000 rpm. In the calculations it was assumed that the degree of sensitivity of the control dimensions (values/tolerances) implies differences in the efficiency values of the tested pumps. Figure 13 shows a comparison of the total efficiency of ten gear pumps *3PWR-SE-32/28-2-776* for $n = 500$ rpm.

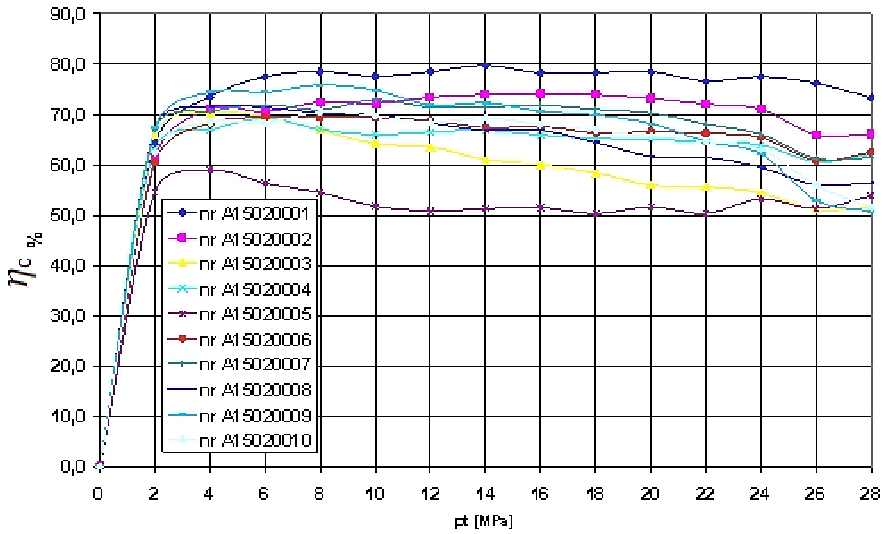


Figure 13. Comparison of total efficiency η_c of *3PWR-SE* pumps for $n = 500$ rpm.

Source: based on own research analysis; Kollek et al., 2015.

Then, for the rotational speed $n = 500$, 800, 1000, 1500 and 2000 the maximum values of the total efficiency η_c of the pumps were determined. A 5% deviation was assumed and for such a range: 95-100% of the maximum efficiency value a set of pumps meeting this criterion was determined. Identification of the influence of the measurement points tolerance on the given efficiency values required application of the Quine - McCluskey algorithm of minimization of partial multi-valued logical functions for the common interval range of all 10 pumps. After applying the multi-valued logic tree method, the control dimensions critical, important, and unimportant were determined for the six details of the pumps in question: a driven gear, a driven gear, a bearing set, a body, a plate, a cover, and a bolt tightening force.

Figure 14 shows the schematic diagram of the assembly drawing of the *3PWR-SE* pump (Deptuła et al., 2017).

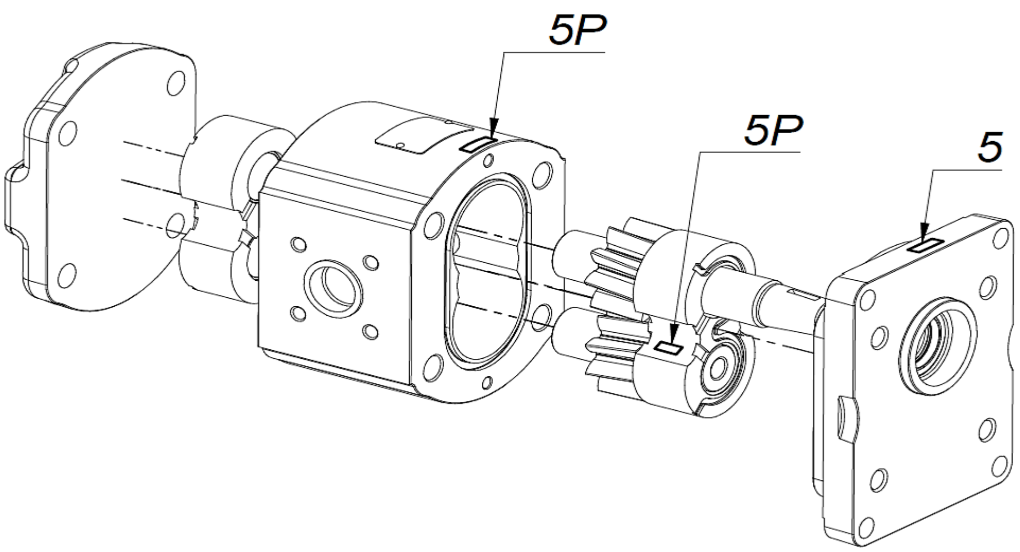
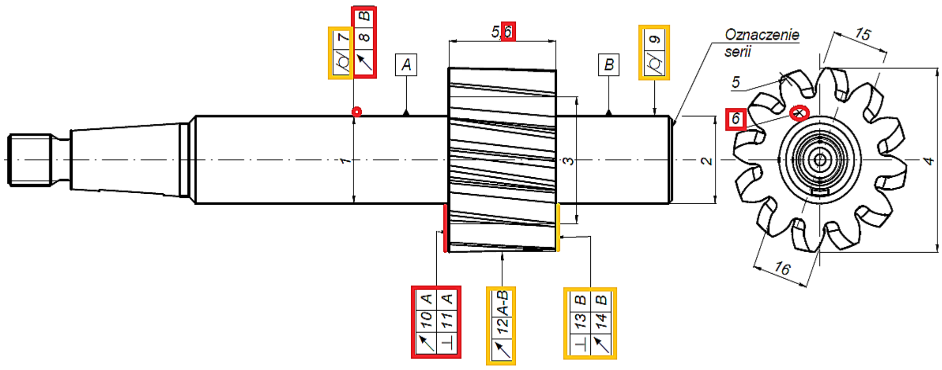


Figure 14. Assembly drawing of tested prototype gear pump *3 PWR*.

Source: Deptuła et al., 2017.

Figure 15 shows the most important and less important control points for the details (active and reactive gears and body assembly):

a)



b)

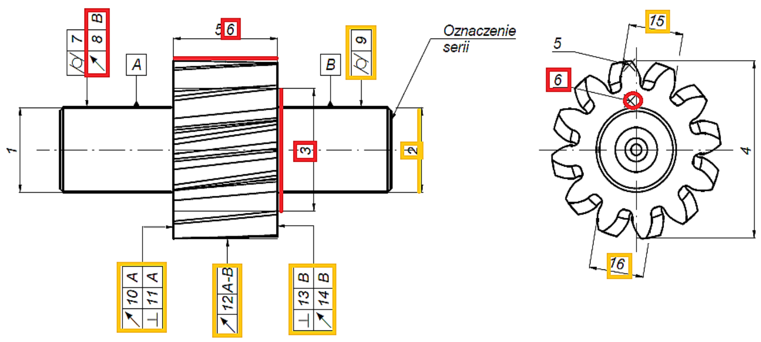


Figure 15. The most important and less important control points for active and reactive gears.

Source: based on own experimental experience.

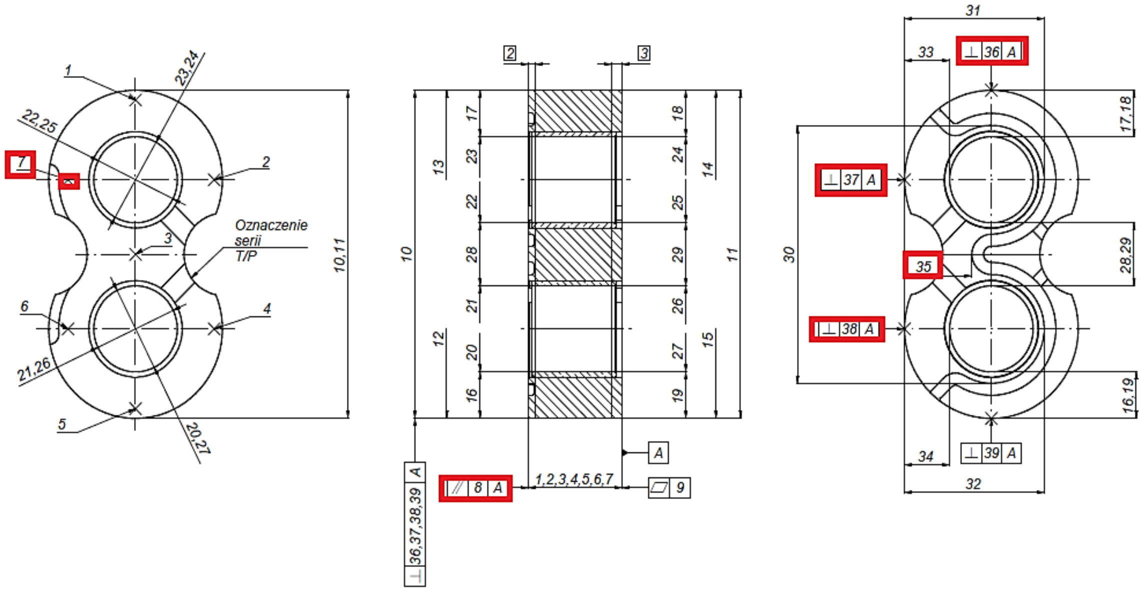


Figure 16. Bearing housing assembly.

Source: based on own experimental experience.

A total of 40 units of model and prototype pumps *2PWR-SE*, *3PWR-SE*, *1PWR-SE*, and *2PW-SEW* were tested. The number 142 control dimensions were evaluated for each pump making a total of 5.680 dimensions. The study of identification of the influence of manufacturing technology of model units by the method of multi-valued logical trees showed that significant (important) dimensions affecting the efficiency of pumps generally repeat in all details regardless of the analyzed group. The most frequently occurring critical dimensions for individual components of the pump are runout of the journal and side surface of the gear ring, perpendicularity of the tooth side surface to the journal (axis of rotation of the wheel) and perpendicularity of the well to the pump face (Figure 17) as well as perpendicularity of the forming outline to one of the bearing thrust planes: flatness of the bearing thrust surface, parallelism of the bearing thrust planes (Figure 18).

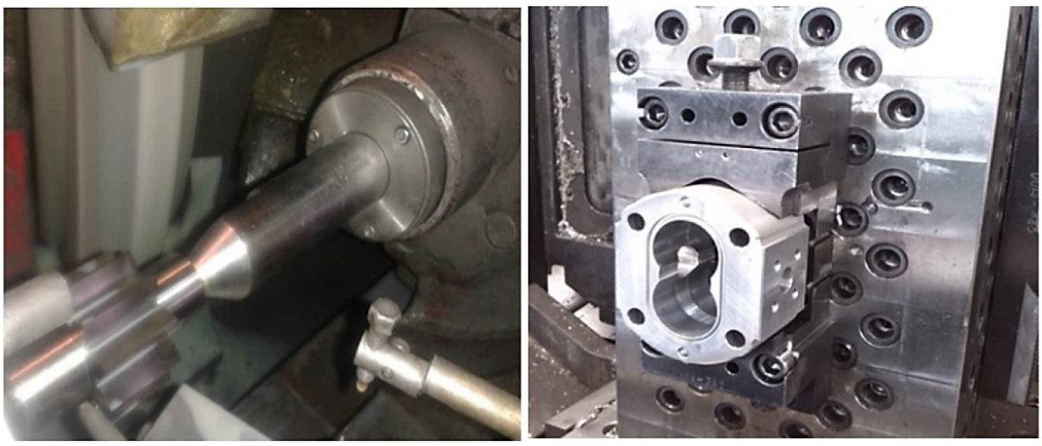


Figure 17. Grinding of journals on the claw grinder - detailing driven and driven gears, clamping of the body made on the CNC machine - pump body.

Source: Osiński, 2017.

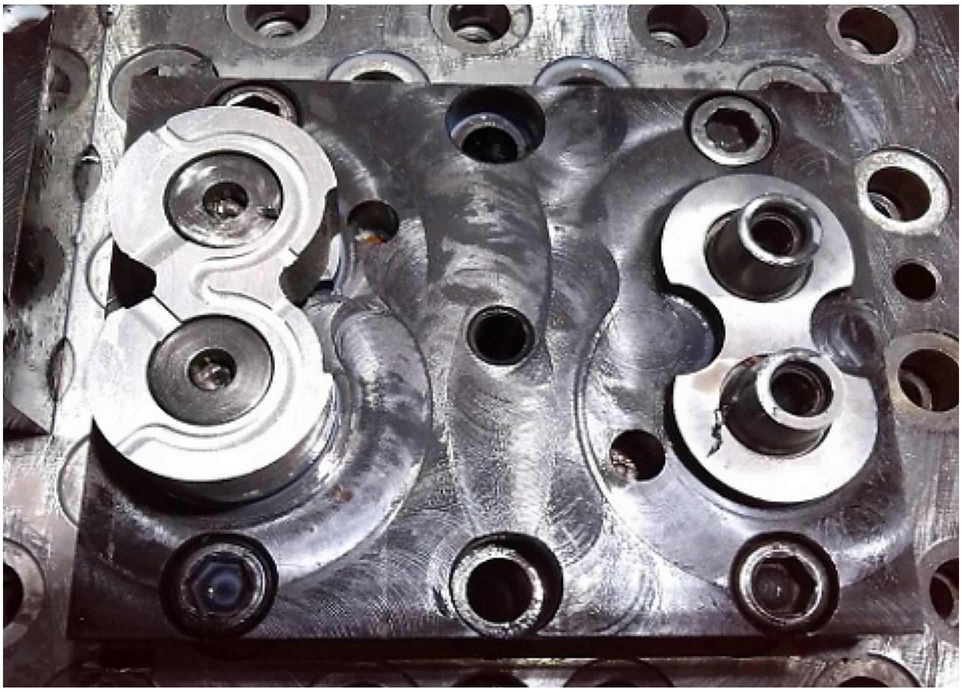


Figure 18. Mounting of plain bearing housing on CNC machine table - detail of bearing housing assembly.

Source: Osiński, 2017.

After the optimization of prototype pump manufacturing technology, less scatter in the efficiency results is observed. Comparing the acoustic characteristics of prototype pumps with ground and chipped wheels (Deptuła et al., 2020) made in backlash - free technology, it turned out that the solution with ground wheels has 3 to 5 dB lower noise emission to the environment. Optimization using multi-valued logic tree methodology resulted in a rational narrowing of dimensional and shape tolerances where necessary and a reduction of accuracy class in minor locations. This ultimately contributed to lower production costs and increased productivity.

6. Results

The developed and adopted methodology for designing gear pumps with the use of multi-valued logic trees concerns two aspects: the object of design - the studied pump and the design process - a sequence of activities with logical ordering. In particular, they are applied in the improvement design, which concerns the existing systems and is oriented at the elimination of the identified deficiencies in the applied solutions and their improvement. The methods are also applicable in basic design.

A general scheme for finding solutions using the multi-valued logic tree method is shown in Figure 19.

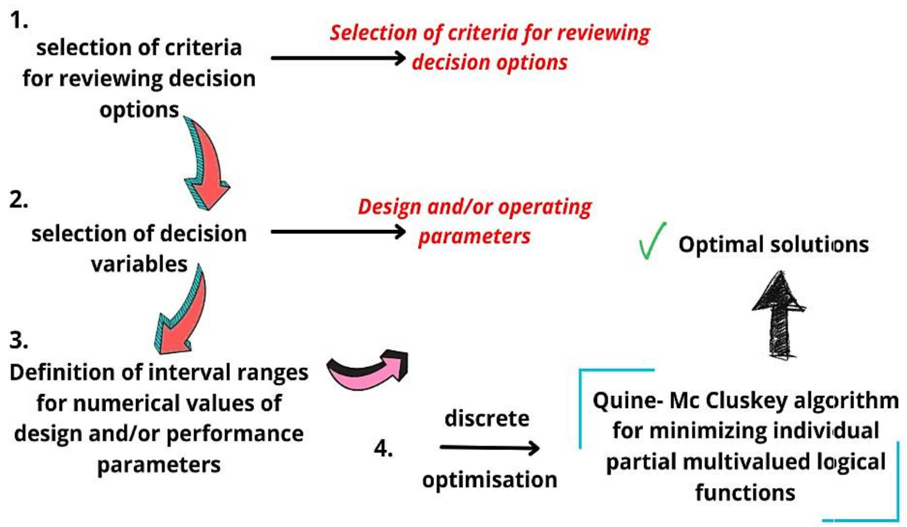


Figure 19. Solution search method using multi-valued logic trees.

Source: based on own research analysis.

The objects of the research were model and prototype pump designs whose tooth outline was optimized using multi-valued logic trees. The optimization process has been carried out taking into account five basic criteria: technological feasibility of the tool, obtaining minimum compression ratio, occurrence of small changes of dynamic forces in the gear, obtaining minimum efficiency pulsation ratio, and ensuring high energy efficiency.

After a positive verification of the wheels printed in polyethylene we started to make them in industrial conditions. The surface of the three-rotor outline was made using the following technologies: ground and chipped (Figure 20). Using logical decision-making structures, an optimization of the technology of processing of elements influencing the total efficiency of the newly-designed unit was also carried out. The analysis of dimensional and shape tolerances ultimately allowed for the selection of control dimensions: critical, important and unimportant. This resulted in rational narrowing of dimensional and shape tolerances where it is necessary and lowering of accuracy class in places of minor importance. Optimization of manufacturing technology contributed to lowering of manufacturing costs and increasing its efficiency.

a) b)

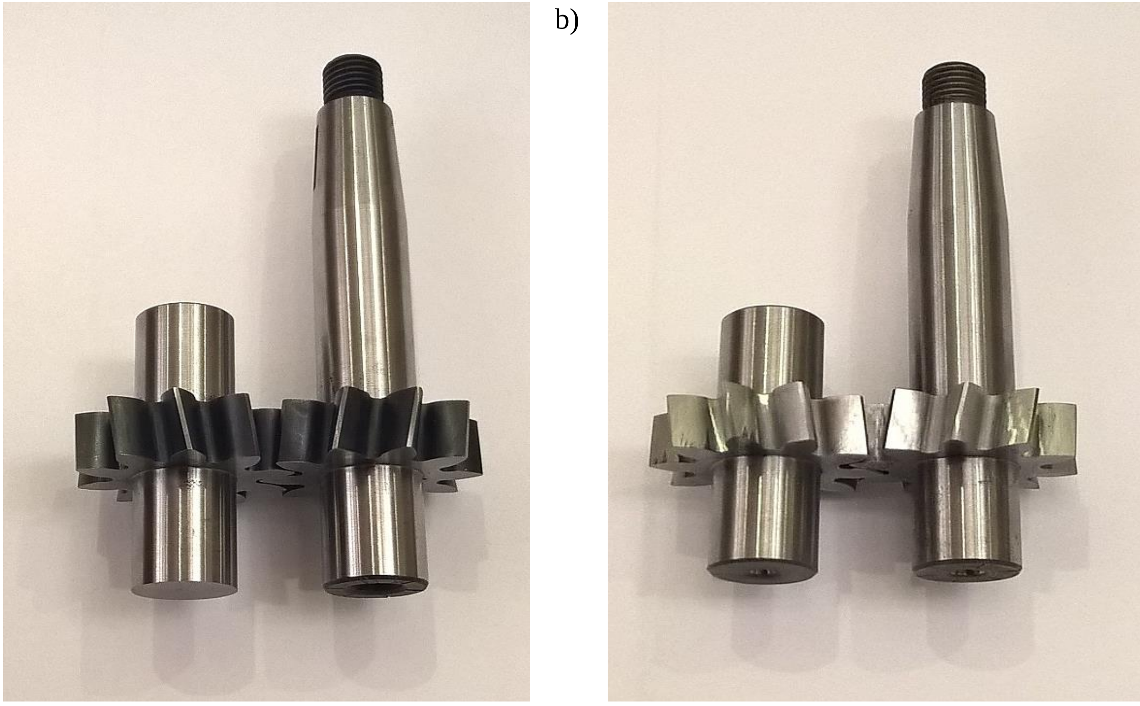


Figure 20. Gears: a) chipped, b) ground.
Source: Osiński, 2017.

After the optimization of the technology of making prototype pumps, smaller scatter in the results of the efficiency run is observed.

7. Conclusions

The paper presents a methodology for designing gear pumps using multi-valued logic trees. The solutions obtained by means of the adopted methodology have been structurally supported by two patent applications and industrial implementation. The most important utilitarian features of the newly developed pump designs include:

- 78.5% reduction in performance pulsation,
- 4 to 5 *dB* reduction in ambient noise emission,
- reduction of sound generating vibrations by approx. 50%,
- increase in total efficiency from 9 to 12%,
- improving the conditions for relieving the congested space by providing a degree of coverage $\boldsymbol{\varepsilon} \approx \mathbf{1}$.
- reducing the compression ratio to 1.

The development of the multi-valued logic tree methodology required, among other things:

- Introduction of multi-valued weighting coefficients for determining the most important design guidelines. This made it possible to apply the method of multi-valued decision trees to solve technological problems, especially in the technological process of gear pump manufacturing.
- The use of systems of logical equations as a formal decision-making description for testing the importance rank of changes in the arithmetic values of such parameters.
- Development of the method of minimization of logical equations of multi-valued systems taking into account different multi-valued decision variables, as a method of optimization of bypass valves and other hydraulic systems.
- Consideration of certain and uncertain information data in optimization of dynamic properties of machine systems.

The developed methodology of multi-valued logic trees and parametric graphs in particular provides a new method for discrete optimization. The analytical description of the object speeds up finding the optimal solution and significantly reduces the cost of the optimization process. The methods for finding the global minimum of the objective function can be divided into two groups. The first group are heuristic methods, which allow to find the global minimum only with certain probability. In which also synthesis and analysis of the problem can be considered (Bamford, 2002). The second group consists of methods that allow finding the global minimum with some known accuracy. At this stage one can recall the method of subjective objective system (Ziv-Av, Reich, 2005) which has applications in many engineering fields (e.g. mechanical engineering), but also non-engineering fields (e.g. design of a new magazine, banking service, and also in the case of studying the effect of wood anisotropy on its mechanical properties - with respect to the effect of scale (Malaga-Tobola et al., 2019).

The multi-valued decision tree method is a new optimization tool classified as an interval optimization method. The estimation of global minimum obtained by the method of compartmental global optimization is correct in every iteration step what gives a possibility to terminate calculations at any time or when assumed structural and design accuracy is achieved. The structuring of the problem described by means of multi-valued logical trees and parametric graphs makes it possible to introduce appropriate formal notations and, in particular, it is even possible to combine complex quantitative and qualitative features of varying degrees of detail according to the principles of a multidimensional morphological array. Moreover, morphological and decision arrays can be analytically and numerically encoded according to the definitions and theorems of the logic of multi-valued decision processes, which enables a variant way of identifying and classifying information in computer terms during the search and modification of solutions in the design process. The analyzed problem, can also be categorized as design methods (Daalhuizen, Cash, 2021) allowing to capture key procedural knowledge, crucial for the design process, practice and education.

The method of optimization of a discrete gear pump, using multi-valued logic trees, is a new approach to the problem. A suitably adapted method of coding selected parameters on multi-valued trees and an algorithm for determining realizable design guidelines for a pump with tooth foot modification, allowed to generate the importance rank of design parameters and optimal changes to improve mechanical, volumetric and total efficiency. The discrete optimization also showed that it is possible to significantly reduce the dynamic load amplitudes in the pump already at the design stage. At the same time, the obtained results initiated further research on optimization of different pump types using multi-valued logic tree methodology.

It is important to note that, while the methods used in this study - such as multi-valued logic trees and finite element analysis (FEA) - enabled effective optimization of gear pump design, certain limitations are present. Primarily, our experiments were conducted in laboratory conditions, which may not fully reflect the diverse operational environments encountered in industrial applications. Additionally, precise modeling of dynamic phenomena such as cavitation and variable loads, which may arise during extended pump operation in changing environments, presents some constraints. The numerical approach we employed is also limited by the assumptions and simplifications inherent to mathematical modeling, which may affect the accuracy of results. Several key areas could guide future research. First, we plan to conduct tests in real-world conditions to assess the performance and durability of prototype gear pumps under varied operational settings, such as high-pressure environments, elevated temperatures, and fluctuating loads. Further advances could be achieved by refining flow modeling, specifically by applying advanced CFD methods to deepen understanding of the effects of dynamic parameters, such as variable load and cavitation, on pump durability and efficiency. Another potential direction involves the development of hybrid models that combine finite element analysis with artificial intelligence-based optimization methods, which could further enhance design precision. Additionally, the incorporation of composite or wear-resistant materials may improve pump durability in demanding conditions. In summary, supplementing our research with real-world testing and implementing advanced modeling and optimization methods could significantly expand the industrial applications of our findings, representing a promising direction for future studies.

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