SILESIAN UNIVERSITY OF TECHNOLOGY PUBLISHING HOUSE

SCIENTIFIC PAPERS OF SILESIAN UNIVERSITY OF TECHNOLOGY 2024
ORGANIZATION AND MANAGEMENT SERIES NO. 201

THE ANALYSIS OF SPATIAL-TEMPORAL DIFFERENCES IN UNEMPLOYMENT RATES IN POLAND BY COUNTIES IN THE YEARS 2019-2023

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Purpose: The aim of the article is to identify the spatial differentiation of changes in the unemployment rate in the county system in Poland in the period 2019-2023. The subject of particular interest is the verification of the hypothesis about the existence of spatial dependencies in changes in the level of unemployment rates between counties considered to be neighboring in terms of a given criterion.
Design/methodology/approach: The validity of using the methods and tools of spatial econometrics to describe unemployment as one of the most important negative socio-economic phenomena is confirmed by numerous empirical analyses, and in the face of dynamic economic and social changes, it does not lose its importance. The analysis used spatial and space-time econometric models. The spatial structure of dependencies between counties was quantified using the common border matrix.
Findings: Research confirms the existence of spatial dependencies in the development of the unemployment rate registered in the counties in Poland.
Practical implications: The practical implication of this study comes from the provision of evidence that when it comes to analyzing processes within specific areas, it is essential to account for the spatial relationships between objects, as these relationships significantly influence the outcomes and dynamics observed.
Originality/value: The originality of the study comes from the tool used, which enables the analysis of processes through the prism of the structure of relationships between the objects they concern.
Keywords: registered unemployment rate, spatial models for time-series and cross-sectional data, spatial interactions, pandemic.
Category of the paper: Research paper.

1. Introduction

Unemployment is a social problem that has materialized in Poland with varying intensity over the last several decades. High unemployment accompanied the transformation process from a centrally planned economy to a market economy in the 1990s. Its significant decline occurred in connection with Poland's accession to the European Union in 2004, and further rapid upward changes were recorded in the second half of 2008, when the first symptoms of the global financial crisis became felt in Poland and the registered unemployment rate amounted to 9 .6 percent. We can talk about a gradual decline in unemployment only from 2013, when the consequences of the European debt crisis began to slowly fade away.

Another wave of the increase in the unemployment rate in Poland was expected at the beginning of 2020 with the outbreak of the Covid-19 pandemic and the introduction of a state of epidemic on March 20. However, the forecasts made at the beginning of the pandemic regarding an increase in unemployment to 7,5-9,9 percent have not been reflected. The differences between the predictions and the actual increase in unemployment recorded at that time can be justified, among others, by the effectiveness of the anti-crisis measures taken to protect jobs, the quality of data on unemployment, or the very definition of the term "unemployed". During the strict lockdown, a smaller percentage of people losing their jobs were able to register at employment offices. Additionally, due to concerns about the upcoming economic crisis, some unemployed people could temporarily suspend their job search, which did not make them unemployed in the strict sense, but economically inactive (Kukołowicz, 2021).

Unemployment as a multidimensional phenomenon is determined by several different factors. According to Phillips' concept, a slower growth in nominal wages is accompanied by an increase in the unemployment rate. In turn, based on Okun's law (1962), it should be expected that each decrease in real gross domestic product by two percentage points will be accompanied by an increase in the unemployment rate by one percentage point. The increase in employment favors the growth of the gross domestic product, which in turn stimulates economic growth while simultaneously leading to a decline in the unemployment rate. Also, factors defining a specific labor market, such as the number of people of working age, professional activity of the population or the number of registered business entities, may stimulate both an increase and a decrease in the unemployment rate in each region. Additionally, the level of investments, the impact of which on the unemployment rate is observed in the long term, or the socio-economic position of areas adjacent to a given region, which may stimulate the phenomenon of migration, should also be indicated as a determinant of the unemployment level.

Migrations are a manifestation of the spatial adaptation of the population living in each region to changing living conditions and are an important factor influencing the spatial differentiation of the unemployment rate observed in Poland. The scale of the mismatch between supply and demand for labor can be demonstrated by the difference between the highest and the lowest value of the unemployment rate registered in counties, which in 2023 amounted to twenty-three percent. The literature on the subject often also points to socio-economic factors, specific to adjacent regions, as an additional factor determining the level of unemployment in each area.

The analysis of unemployment rates is a well-recognized direction of empirical analyzes carried out using various tools. These include, for example: Muller-Frączek, Pietrzak, 2012; Litwińska, 2012; Cracolici et al., 2009; Pereira et al., 2017.

2. The aim and the scope of the analysis

The aim of the article is a spatial analysis of the unemployment rate registered in Poland in 2019-2023. The central issue of the conducted empirical analysis is the answer to the question whether, due to the Covid-19 pandemic, there have been changes in the spatial distribution of the relationship between the unemployment rates of neighboring regions. A research hypothesis was formulated according to which, in the analyzed periods, there was a change in spatial dependencies in the development of registered unemployment rates in neighboring regions. To verify it, an approach based on the estimation of spatial models for time-series and cross-sectional data was used.

The empirical study was conducted using monthly data obtained from the Central Statistical Office. The analysis horizon was divided into three sub-periods falling respectively before the outbreak of the pandemic, then after the announcement of the state of epidemic in Poland and after the lifting of previously imposed restrictions, and the argument in favor of estimating models separately for each of the sub-periods is, among others, the change in the trend from a downward to an upward research horizon, and then another decline in the registered unemployment rate observed at the end of the analyzed period.

The structure of the article is as follows. The second part of the article focused on methodological issues, explaining the essence of spatial models, and indicating their variants used in further analysis. Chapter three contains a discussion of the results of the empirical study. The summary synthesizes the main observations and indicates directions of analyzes and considerations worth undertaking in the future regarding the analyzed problem.

3. Methodology

The spatial dimension of the economic process is said to be when location and neighborhood influence the way interactions and patterns of socio-economic changes are shaped. Haining (2003) distinguishes four types of spatial processes: diffusion, exchange and transfer, interactions, and dispersion. Diffusion is said to occur when a feature introduced into a population remains permanently in an individual originating from it. Exchange and transfer involve different locations becoming similar in terms of a specific feature because of the flow of goods and services between them. When the results of spatial processes in one location begin to determine their shape in other areas, interactions between these areas are said to occur.

Since the registered unemployment rate is not independent of its location, spatial models for time-series and cross-sectional data were used to model this phenomenon. Considering the space factor allows us to verify whether mutual interactions of neighboring regions can lead to common patterns in the development of the level of this phenomenon.

The advantage of the approach used is, among others, the ability to consider elements of dynamics in the estimated models, which is important because there is a possibility of delayed reactions with which counties influence each other in terms of the level of the considered variable.

One of the basic variants of models that consider spatial interactions between areas is the spatial error model of the following form:

$$B_{it} = \beta_0 + \beta_1 B_{it-1} + \eta_i, \eta_i = \lambda \sum_{j \neq i} w_{ij,t} \eta_{jt} + \varepsilon_{it}. \tag{1}$$

The spatial error model assumes that the source of spatial interactions is a spatially correlated random component. This means that interactions between objects are caused by factors not included in the model. This scenario seems even more likely when we realize the multitude of factors influencing the unemployment rate.

To consider the existence of several sources of spatial dependencies, the SARAR (Spatial Autoregressive with Autoregressive Disturbances) model, also called the SAC (Spatial Autocorrelation with Correction of Error) model, as well as the general spatial model SGM (Spatial General Model) were used to model the unemployment rate (see: Kelejian, Prucha, 2008; Suchecki, 2010), with the following form:

$$B_{it} = \beta_0 + \beta_1 B_{it-1} + \rho \sum_{j \neq i} w_{ij,t} B_{jt} + \phi \sum_{j \neq i} w_{ij,t} B_{jt-1} + \eta_i, \eta_i = \lambda \sum_{j \neq i} w_{ij,t} \eta_{jt} + \varepsilon_{it}. \tag{2}$$

In models (1) and (2), B_{it} denotes the monthly unemployment rate registered in the i-th county at time t; B_{it-1} is a time-delayed dependent variable which allows for taking into account elements of the dynamics of the analyzed process in its modeling; $w_{ij,t} B_{jt}$ reflects the unemployment rate but in neighboring county in terms of the adopted criterion, and this is the so-called spatial shift; $w_{ij,t} B_{jt-1}$ is the time-spatial delay, i.e. the unemployment rate in period $t-1$ in neighboring locations; $w_{ij,t} \eta_{jt}$is in turn, a spatially correlated random component, which is the second source of spatial interactions in the second model.

Particular attention should be paid to the interpretation of parameters related to spatially and spatiotemporally shifted explanatory variables and the spatially correlated random component. The parameter ρ informs about the strength of spatial interactions between counties and reflects simultaneous changes in the unemployment rates of counties considered to be neighboring in terms of a given criterion. The value of the ϕ parameter standing for the temporally and spatially lagged dependent variable indicates the impact of the unemployment rate from the previous period in counties considered to be neighboring a given county on its current unemployment rate. In turn, the λ parameter reflects the autoregressive, spatial error component. There is a possibility that there is a variable with spatial impact but not included in the model. Hence, spatial autocorrelation may be detected in its residuals.

The parameters $w_{ij,t}$ in models (1) and (2) reflect the structure of mutual spatial dependencies between counties and come from the neighborhood matrix **C** used to formally map interactions related to spatial location. This study used the criterion of having a common border as the definition of neighborhood. Defining it in this way leads to the creation of a first-order contingency matrix that is symmetrical and quadratic, with the number of columns and rows corresponding to the number of counties. The elements c_{ij} of matrix **C** have the form (Suchecki, 2010):

$$\begin{cases} c_{ij} = 1, when\ there\ is\ a\ common\ border\ between\ the\ i-th\ and\ j-th\ county, \\ c_{ij} = 0, when\ there\ is\ no\ common\ border\ between\ counties. \end{cases}$$

so:

$$\boldsymbol{C} = \left[c_{ij}\right]_{N\times N}. \tag{3}$$

The matrix constructed in this way was subjected to the procedure of row-by-row standardization to unity, transforming its elements according to the formula:

$$w_{ij} = \frac{c_{ij}}{\sum_{j=1}^{N} c_{ij}}. \tag{4}$$

As a result of the above transformations, we will obtain a matrix **W** such that:

$$\mathbf{W} = \left[w_{ij}\right]_{N\times N}, \wedge_i \sum_{j=1}^{N} w_{ij} = 1. \tag{5}$$

Initial testing of the validity of introducing spatial effects into the model was carried out using the global Moran's *I* statistic, which enables verification of the occurrence of global spatial autocorrelation, based on the scheme of spatial connections described by the weight matrix **W**. Therefore, having a standardized row weight matrix **W** and observations of the analyzed variable $Z(\boldsymbol{s}_i)$, i.e., unemployment rates registered in individual locations $\boldsymbol{s}_i$, the value of the *I* statistic is given by the following formula (see e.g. Schabenberger, Gotway, 2005):

$$I = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}[z(\boldsymbol{s}_i)-\bar{z}][z(\boldsymbol{s}_j)-\bar{z}]}{\sum_{i=1}^{N}[z(\boldsymbol{s}_i)-\bar{z}]^2} = \frac{\boldsymbol{z}^T W \boldsymbol{z}}{\boldsymbol{z}^T \boldsymbol{z}}, \tag{6}$$

where:

$z(\boldsymbol{s}_i)$ − value of the registered unemployment rate $Z(\boldsymbol{s}_i)$ in the *i*-th county;

$\bar{z}$ −average value of unemployment rates;

$\boldsymbol{z}$ −column vector with elements $z_i = z(\boldsymbol{s}_i) - \bar{z}$;

$S_0 = \sum_i^N \sum_j^N w_{ij}^*$ −sum of all elements of the weight matrix.

Testing the statistical significance of Moran's *I* statistics comes down to verifying the null hypothesis about the random distribution of the values of the analyzed variable in individual locations, i.e. the absence of spatial autocorrelation (Cliff, Ord, 1973, 1981; Suchecki, 2010).

For this purpose, the normalized Z_I statistic with a normal distribution with the expected value equal to zero and unit variance is used, i.e.:

$$Z_I = \frac{I - E(I)}{\sqrt{Var(I)}} \sim N(0,1), \tag{7}$$

where:

$$E(I) = -\frac{1}{N-1}, \tag{8}$$

$$Var(I) = \frac{N^2 S_1 - N S_2 + 3 S_0^2}{(N^2-1) S_0^2} - \frac{1}{(N-1)^2}, \tag{9}$$

wherein:

$S_0 = \sum_i \sum_j w_{ij}$,

$S_1 = \frac{1}{2} \sum_i \sum_j (w_{ij} + w_{ji})^2$,

$S_2 = \sum_i \left(\sum_j w_{ij} + \sum_j w_{ji} \right)^2$.

If spatial autocorrelation does not occur in relation to the analyzed phenomenon, the value $I \approx -\frac{1}{N-1}$, $Z_I \approx 0$. Otherwise, positive autocorrelation can occur when:

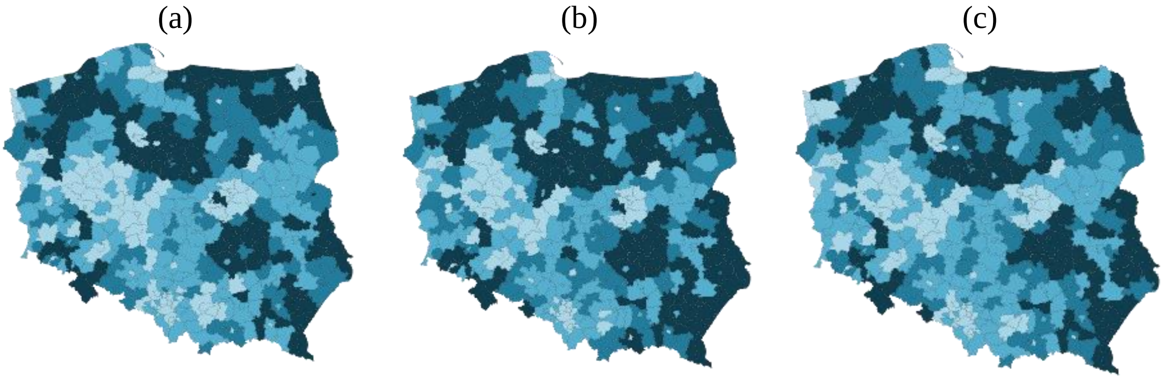
$$I > -\frac{1}{N-1}, Z_I > 0, \tag{10}$$

or negative autocorrelation when:

$$I < -\frac{1}{N-1}, Z_I < 0. \tag{11}$$

4. Results of the empirical analysis

Even though the labor market in Poland consists of many local markets and the spatial scope of counties is small, it is characterized by rather low spatial mobility of employees. Although this regularity adversely affects potential spatial interactions between neighboring counties, considering spatial effects related to the location of areas in modeling the unemployment rate is a quite commonly explored direction of analysis. There is an assumption, observable in Figure 1, regarding the formation of spatial clusters of areas with similar values of the registered unemployment rate.



* The division was made using positional measures: dark blue – very high values; blue – high values; light blue – medium values; very light blue color – low values.

Figure 1. Spatial distribution of the registered unemployment rate in the subperiods: (a) 01/2019-02/2020, (b) 03/2020-05/2022 and (c) 06/2022-12/2023

Source: author's own study.

The largest number of counties in the group with the highest level of registered unemployment (dark blue color) were classified in the second subperiod, which coincided with the period of strong expansion of the Covid-19 pandemic, resulting with an implementation of further restrictions aimed at preventing the spread of the virus. It is worth noting that the counties from the north-east area of Poland had the relatively highest unemployment rate registered in this period so the spatial trend can be observed.

By analyzing the Figure 1(c) it can be observed that the number of counties in northern Poland with the highest unemployment rate has decreased. Counties in this area have neutralized the effects of the pandemic better than counties located in the south-east.

A clear clustering of objects with the average height of the analyzed variable can also be seen in Figure 1(a). These clusters cover the whole of Poland but with varying intensity.

A preliminary analysis using visualization on maps, confirms the validity of considering the unemployment rate in terms of spatial processes.

The local nature of labor markets, the fact that cooperation is more common between neighboring counties rather than between areas located at a considerable distance from each other and the short distances that job seekers usually travel were an argument standing in favor of using a neighborhood matrix based on a common border.

The analysis began with checking the stationarity of a series of an unemployment rate. Then the classical models for each of the analyzed subperiods were built. The analysis of the residuals of the classical models allows to conclude about the presence or absence of spatial effects. The results of their estimation and verification are presented in Table 1.

Table 1.
Results of estimation and verification of the classical model in the analyzed subperiods

Parameter	**Subperiods**		
	January 2019 – February 2020	March 2020 – May 2022	June 2022 – December 2023
Const.	0,021 (0,015)	0,033 (0,029)	0,132 (0,111)
B_{t-1}	0,989 (0,000)	1,221 (0,000)	1,078 (0,000)
Statistics			
R^2	0,519	0,872	0,549
Test F	102,991 (0,000)	99,212 (0,000)	57,100 (0,000)
Moran *I*	0,055 (0,000)	0,069 (0,000)	0,044 (0,001)
LM			
LM_{err}	31,707 (0,000)	72,651 (0,000)	22,662 (0,002)
LM_{lag}	0,141 (0,707)	0,155 (0,891)	0,065 (0,678)
RLM_{err}	31,589 (0,000)	44,759 (0,000)	27,443 (0,031)
RLM_{lag}	0,023 (0,880)	0,095 (0,990)	0,261 (0,456)

Note: The W matrix is used here in the Moran test and LM tests.

Source: author's own study.

In this study, it is assumed that the registered unemployment rates of counties located in their neighborhood are correlated with each other, which is identified with the occurrence of spatial autocorrelation. As it can be concluded based on values of Moran's I statistics the null hypothesis should be rejected in favor of the alternative one according to which spatial autocorrelation occurs, in all three subperiods. This argument indicates the need to depart from the classical approach. The non-random nature of the residuals of the estimated models indicates the occurrence of certain important variables, although not included in the model, or properties that influence analyzed process.

It is worth noting the statistical significance of the time-lagged registered unemployment rate, which means that the current recorded value is influenced by the values of this variable recorded in the previous month, due to the monthly aggregation of data used in the study.

To propose an alternative to the classical approach, Lagrange multiplier tests and their robust versions were used. They consider the construction of a spatial autoregressive model and a spatial error model as an alternative. In each of the analyzed subperiods, their results suggest the validity of the construction of the spatial error model, in which the source of spatial interactions is a spatially correlated random component. The results of estimation and verification these models are presented in Table 2.

The values of Moran's *I* statistics indicates that autocorrelation of residuals does not occur, and therefore extending the models with spatial dependencies was the right approach. What is particularly important the parameter λ relating to the spatially correlated random component shows statistical significance in all three analyzed subperiods too and the significance of the spatial effects is additionally confirmed by the values of the F statistics. The inclusion of spatial effects in the modeling resulted in a significant decrease in the residual variance of the model, as the results of Chow test indicate.

Table 2.
Results of estimation and verification of the spatial error model in the analyzed subperiods

Parameter	**Subperiods**		
	January 2019 – February 2020	March 2020 – May 2022	June 2022 – December 2023
Const.	0,227 (0,015)	0,033 (0,029)	0,132 (0,111)
B_{t-1}	0,887 (0,000)	1,221 (0,000)	1,078 (0,000)
λ	0,230 (0,001)	0,777 (0,000)	0,645 (0,001)
Statistics			
Wald statistic	3,221 (0,012)	4,741 (0,003)	1,802 (0,007)
Log Likelyhood	102,991 (0,000)	99,212 (0,000)	57,100 (0,000)
Spatial effects F-Chow	2,321 (0,004)	7,213 (0,000)	5,435 (0,011)
Moran I	0,005 (0,398)	0,011 (0,928)	0,992 (0,234)

Source: author’s own study.

In each of the analyzed periods, the value of the lambda parameter is also important, which indicates the existence of non-model sources of spatial dependencies between counties. Despite the confirmation of the significance of spatial effects using the Chow test, the highest level of significance of these effects was observed in the second subperiod. An additional explanatory variable in the form of a time-lagged dependent variable is also statistically significant.

The last variant of the estimated models assumes the two sources of spatial dependencies. On the one hand, it allows us to assess the impact of the unemployment rates of counties considered to be neighboring on the registered unemployment rate recorded in each county, but it also considers the presence of a spatially correlated random component which reflects out-of-model spatial patterns influencing the explained variable. The estimated values of the SGM models parameters as well as the results of their verification for all analyzed subperiods are presented in Table 3.

Table 3.
Results of estimation and verification of the general spatial model in the analyzed subperiods

Parameter	Subperiods		
	January 2019 – February 2020	March 2020 – May 2022	June 2022 – December 2023
Const.	0,238 (0,115)	0,133 (0,079)	0,321 (0,141)
B_{t-1}	0,897 (0,000)	2,221 (0,000)	1,184 (0,000)
$B_{t-1_{lag}}$	-1,098 (0,037)	1,275 (0,222)	-0,222 (0,003)
ρ	0,777 (0,004)	0,897 (0,041)	1,022 (0,000)
λ	1,370 (0,032)	2,897 (0,000)	1,777 (0,001)
Statistics			
Wald Statictic	2,901 (0,002)	6,734 (0,023)	1,892 (0,017)
Log Likelyhood	102,991 (0,000)	99,212 (0,000)	57,100 (0,000)
Spatial effects F-Chow	3,459 (0,002)	6,547 (0,000)	2,311 (0,041)
Moran *I*	0,205 (0,498)	0,718 (0,528)	0,999 (0,634)

Source: author's own study.

None of the models considered is characterized by spatial autocorrelation of residuals, which is confirmed by the results of Moran's I statistics. The validity of the model form used was also confirmed by the significance of the Wald statistic. Attention should be paid to the statistical significance of the λ parameter, which indicates the existence of non-model factors influencing changes in the explained variable and the spatial autoregression parameter ρ.

The parameter ρ informs about the strength of spatial interactions between the analyzed counties. Its value reflects the average impact of the unemployment rate registered for neighboring objects in terms of the neighborhood criterion selected at the model specification stage on its value in the *i*-th county. The highest level of significance the parameter ρ has in the first and second subperiods. During the period of strong expansion of the pandemic and the implementation of subsequent restrictions i.e., between March 2020 and May 2022, the significance of the spatial autoregression parameter ρ is only at the level of 0.05.

In the considered model form, the so called general spatial model, it is assumed that there is more than two sources of spatial dependencies between objects so the model include an additional spatially shifted explanatory variable. Assuming that there are feedback loops between counties and reactions are delayed, it is worth considering whether the unemployment rate from a month ago in counties considered to be neighboring has an impact on the observed unemployment rate in a given county.

In the model estimated for the period before the pandemic and after the end of the epidemiological threat, the unemployment rate variable in the previous period as well as the time and space delay of this variable are statistically significant. This means that the unemployment rate registered in a given county is influenced not only by the unemployment rate recorded in the neighboring county, but also by its value from the previous month. However, it is worth paying attention to the opposite signs of the coefficients for these variables. One possible explanation for this regularity may be the fact of compensating changes in the unemployment rate in a given area with changes in the direction opposite to those recorded in regions considered neighboring.

Estimation and verification of both models demonstrate the existence of spatial dependencies in unemployment rate disparities across Polish counties. During the peak of the pandemic, the dynamics of these relationships slightly weakened, likely due to the freezing of local labor markets. Overall, however, spatial autocorrelation is evident across the subperiods studied. There is no evidence to reject the hypothesis formulated in the beginning of the study.

The practical implication of this study comes from the provision of evidence that when it comes to analyzing processes within specific areas, it is essential to account for the spatial relationships between objects, as these relationships significantly influence the outcomes and dynamics observed. Ignoring the spatial aspect of dependencies can lead to incomplete or inaccurate conclusions. The study underscores the importance of incorporating spatially correlated variables alongside traditional idiosyncratic factors when it comes to analyze the differences in unemployment rate.

5. Summary

The conclusions of the study are largely consistent with the results of empirical analyzes carried out so far, considering spatial dependencies in the analysis of the unemployment rate during the period of pandemic.

The use of the spatial error model and the generalized spatial model was justified by the statistical significance of parameters reflecting spatial dependencies included in models with diverse sources of origin. This fact proves the incomplete specification of the estimated classical models.

The study confirmed that the level of unemployment registered in a given county depends not only on the value of this variable in regions considered to be neighboring, but also its time lag.

Nevertheless, the hypothesis formulated in the introduction of the study was not confirmed.

It is worth considering the construction of spatial models taking into account additional regressors in the form of additional explanatory variables of the model, which is partly suggested by the importance of the spatial error parameter in the estimated models.

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