SILESIAN UNIVERSITY OF TECHNOLOGY PUBLISHING HOUSE

SCIENTIFIC PAPERS OF SILESIAN UNIVERSITY OF TECHNOLOGY 2023
ORGANIZATION AND MANAGEMENT SERIES NO. 186

BUSINESS DEMOGRAPHY IN POLAND: A DSGE APPROACH

Paweł ZAJĄC

AGH University of Krakow; pzajac@agh.edu.pl, ORCID: 0000-0003-4343-3458

Purpose: This article explores the relationship between macroeconomic factors and business demography in the Poland in years 2012-2023. Recent events, such as the COVID-19 pandemic and the war in Ukraine, have introduced unique challenges to the Polish economy. Understanding these dynamics is essential for informed decision making.
Design/methodology/approach: The objectives of the paper are achieved using the DSGE approach with fixed number of firms. In the given model, the firm creation process is perceived in a procyclical way using the sunk entry cost mechanism.
Findings: Key findings include the impact of negative productivity shocks on new firms, leading to increased costs and reduced investment attractiveness, resulting in a decline in the total number of companies. In addition, increased public consumption drives up production levels, affecting interest rates and new business opportunities. Technological shocks affect prices and inflation, influencing the number of new companies. Tightening monetary policy impacts capital returns and employment levels. This stimulates the emergence of new firms.
Research limitations/implications: Research limitations are strictly related to the limitation of the tool used. Among the limitations of DSGE models are simplifying assumptions (including perfect rationality, representative agents, and complete markets), lack of detail in the financial sector detail, homogeneity between agents, exogeneity of shocks, calibration, and estimation challenges. Despite these limitations, DSGE models have become a cornerstone of modern macroeconomic analysis and policy evaluation.
Social implications: The findings have important policy implications. Policymakers should consider the potential consequences of their decisions on the business environment, especially in terms of labor market regulations, public spending, and monetary policy.
Originality/value: There are no or few research papers concerning business demography in Poland using the DSGE approach.

Keywords: business demography, DSGE, procyclical entry.

Category of the paper: Research paper.

1. Introduction

Dynamic Stochastic General Equilibrium models (DSGE) were created in response to the need to build relatively simple models whose task is to describe the economy. The standard DSGE class model assumes an unlimited number of enterprises operating on the market and at the same time neglects the impact of the process of establishing and liquidating companies on changes in business cycles (Christiano et al., 2018). Meanwhile, empirical research conducted on US data shows four facts. First, the number of active companies is strongly dependent on the phases of the business cycle. Second, the number of newly established companies explains approximately 20% of quarterly changes in the labor market. Third, Jaravel's (2019) research offers proof that the act of entering a market has an impact on how prices are determined. Fourth, firms less than five years old produce about half of the economy's total output (Colciago, Etro, 2010; Davis et al., 2002; Bernard et al., 2010). Therefore, numerous theoretical works have been written to describe these relationships as best as possible (Casares et al., 2020; Obstbaum et al., 2023). The model proposed by Bilbie et al. (2012) enjoys the greatest recognition among researchers who assume the existence of a finite number of companies on the market. This model is characterized by a procyclical number of new enterprises, which is based on the so-called sunk entry costs for new firms, i.e., the assumption that in the first period of the company's existence, the profits of the company are equal to the costs of starting the business. This mechanism can also be understood as meaning that entrepreneurs decide to start a new business when the value of the new company is equal to the costs of its establishment.

Considering the relationship between macroeconomic variables and the dynamics of the creation and liquidation of companies using DSGE class models provides new opportunities to study the relationships between entrepreneurs and the economy. After appropriate scaling of the model parameters to the realities of Poland, it becomes possible to conduct research that will illustrate the impact of factors such as changes in the labor market, public expenditure, technological shocks, changes in the nominal interest rate, or reducing the costs of starting a business on the demography of enterprises in Poland.

2. The model

Dynamic Stochastic General Equilibrium (DSGE) models are a class of macroeconomic models that play a central role in understanding and analyzing the dynamics of modern economies. DSGE models are grounded in microeconomic principles, with agents (such as households, firms, and the government) making decisions based on optimizing behavior. Agents make decisions today based on their expectations of the future, and these decisions, in turn, affect future outcomes. DSGE models emphasize the importance of general equilibrium, which means that all markets in the economy clear simultaneously. This ensures that supply equals demand in every market, including goods, labor, and financial markets, which is a foundational concept in economic theory. These microeconomic foundations are used to derive the aggregate behavior of the economy.

2.1. Including consumption and prices in the model

Aggregated consumption and price indices

Aggregated consumption C_t depends on the consumption of individual goods $c_{j,t}$, which are indexed using j contained in the set N_t, i.e., the set of goods available at time t. Consumption is expressed using the Dixit-Stiglitz aggregator (1977), which has the CES function form:

$$C_t = \left(\int_0^{N_t} c_{j,t}^{\frac{\theta-1}{\theta}} \, dj \right)^{\frac{\theta}{\theta-1}}, \tag{1}$$

where the parameter $\theta > 1$ stands for the elasticity of substitution between goods.

Let P_t stands for aggregate price index for goods available on the market at time t, and let $p_{j,t}$ determine the price of good j at time t. The desired property here is that $P_t C_t = \int_0^{N_t} p_{j,t} c_{j,t} \, dj$. Therefore, it can be shown that:

$$P_t = \left(\int_0^{N_t} p_{j,t}^{1-\theta} \, dj \right)^{\frac{1}{1-\theta}}. \tag{2}$$

Defining P_t this way leads to a formula determining the demand of individual households for good j (Gali, 2008):

$$c_{j,t} = \rho_{j,t}^{-\theta} \cdot C_t, \tag{3}$$

where:

$$\rho_{j,t} = \frac{p_{j,t}}{P_t} \tag{4}$$

denotes the relative price of the good j at time t (the price of the good divided by the price index).

Types of consumption

The presented model considers the division of aggregate consumption C_t into private consumption $C_{P,t}$ and public consumption $C_{G,t}$. According to empirical research, public consumption is an important component of overall consumer spending in the economy, and its changes significantly affect the behavior of other economic indicators (Blanchard, Perotti, 2002). It is also assumed that the consumer preferences of the public sphere are the same as the consumer preferences of households. Therefore, aggregate consumption in an economy is the sum of private consumption and public consumption:

$$C_t = C_{P,t} + C_{G,t}. \tag{5}$$

Price stickiness

Each company produces only a single good and operates as a monopoly. Therefore, N_t, that is, the number of goods available on the market at time t, is also the number of existing companies. Moreover, the existence of price rigidity is assumed, the dynamics of which is described by the mechanism of Calvo (1983). According to this mechanism, in any period, each existing company can (but does not have to) change the good price of the produced with probability $1 - \Theta$, where $\Theta \in [0,1]$.

Enterprises that can change prices are divided into anticipatory and adaptive enterprises. Anticipatory firms are those that price the good they produce in a way that maximizes discounted profit, while adaptive firms change the price based on past inflation. This value is indexed to the expected price duration, which results in the same (on average) price level for all enterprises operating in the economy.

Let Ξ denote the probability that a firm that can change its price is adaptive. Equation (2) can be transformed into:

$$(\Pi_t)^{1-\theta} = (1-\Theta)(1-\Xi)N_t\left(\frac{p_t^*}{P_{t-1}}\right)^{1-\theta} + (1-\Theta)\Xi\frac{N_t}{N_{t-1}}(\Pi_{t-1})^{\frac{1}{1-\Theta}} + \Theta\frac{N_t}{N_{t-1}}. \quad (6)$$

Formula (6) allows you to determine the optimal price and the equilibrium price level.

2.2. Firms

Production

Companies set a price to make a profit. Let $y_{j,t}$ denote the production volume of good j, and let the aggregate production in the entire economy be denoted as Y_t, then:

$$Y_t = \left(\int_0^{N_t} y_{j,t}^{\frac{\theta-1}{\theta}}\, dj\right)^{\frac{\theta}{\theta-1}}. \quad (7)$$

This leads to a relationship between the production volume of good j and aggregate production using the relative price of good j:

$$y_{j,t} = \rho_{j,t}{}^{-\theta} \cdot Y_t. \quad (8)$$

The company's task is to optimally set the price for the produced good using the available physical capital resources $k_{j,t}$, the amount of employment H_t, and labor $l_{j,t}$. In the production process, enterprises use a two-factor Cobb-Douglas production function:

$$y_{j,t} = A\left(k_{j,t}\right)^{\alpha}\left(H_t l_{j,t}\right)^{1-\alpha}, \quad (9)$$

where A stands for the technology, while α is a parameter describing the elasticity of production with respect to factors, with $0 \le \alpha \le 1$.

Companies optimize values of production factors in two stages. The first stage is to determine the optimal price of manufactured goods and the quality of production. In the second stage, companies take the wage w_t and the return on capital r_t as data and determine the demand for labor and physical capital that will minimize production costs.

Minimization of costs

Firms minimize costs at time t:

$$\min_{(l_{j,t},k_{j,t})} Cost_t = w_t H_t l_{j,t} + r_t k_{j,t}, \tag{10}$$

with production technology:

$$y_{j,t} = A\left(k_{j,t}\right)^{\alpha}\left(H_t l_{j,t}\right)^{1-\alpha} - \Phi. \tag{11}$$

This leads to the Lagrange function:

$$\min_{(l_{j,t},k_{j,t})} Cost_t = w_t H_t l_{j,t} + r_t k_{j,t} - \lambda_{1,t}\left[A\left(k_{j,t}\right)^{\alpha}\left(H_t l_{j,t}\right)^{1-\alpha} - \Phi - y_{j,t}\right].$$

The first-order conditions for firms are given as:

$$\begin{aligned} \frac{\partial Cost_t}{\partial k_{j,t}}: &\quad r_t - \lambda_{1,t}\alpha A\left(k_{j,t}\right)^{\alpha-1}\left(H_t l_{j,t}\right)^{1-\alpha} = 0, \\ \frac{\partial Cost_t}{\partial l_{j,t}}: &\quad w_t H_t - \lambda_{1,t}(1-\alpha)A\left(k_{j,t}\right)^{\alpha}(H_t)^{1-\alpha}\left(l_{j,t}\right)^{-\alpha} = 0. \end{aligned} \tag{12}$$

The Lagrange multiplier is related to technological constraints and is assumed to measure the nominal marginal cost cm_t. Hence the first order conditions have the form:

$$r_t = \alpha cm_t \frac{y_{j,t}}{k_{j,t}}, \tag{13}$$

$$w_t = (1-\alpha)cm_t \frac{y_{j,t}}{H_t l_{j,t}}, \tag{14}$$

and as a result:

$$w_t H_t l_{j,t} + r_t k_{j,t} = cm_t y_{j,t}. \tag{15}$$

Price setting

In the first stage, companies set prices. The optimal price p_t^* is obtained as the quantity that maximizes the weighted, discounted, expected stream of profits:

$$\max_{(p_t^*)} E_t\left\{\sum_{k=0}^{\infty}(\Theta\beta)^k\left[\rho_{j,t}y_{j,t+k} - \left(w_{t+k}H_{t+k}l_{j,t+k} + r_{t+k}k_{j,t+k}\right)\right]\right\}, \tag{16}$$

where:

- E_t stands for expected value at time t,
- $(\beta)^k$ is a discount factor,
- $(\Theta)^k$ is the weight corresponding to the probability that the price cannot be changed for k periods,
- $E_t\{\rho_{j,t}y_{j,t+k} - \left(w_{t+k}H_{t+k}l_{j,t+k} + r_{t+k}k_{j,t+k}\right)\}$ is the firm's profit at period $t+k$, expected at time t.

Substituting equations (15), (10), (4) into (16) and assuming that $p_t^* = p_{j,t}$, the following was obtained:

$$p_t^* = \left(\frac{\theta}{\theta-1}\right)\frac{\sum_{k=0}^{\infty}(\Theta\beta)^k\left[Y_{t+k}(P_{t+k})^{\theta}cm_{t+k}\right]}{\sum_{k=0}^{\infty}(\Theta\beta)^k\left[Y_{t+k}(P_{t+k})^{\theta-1}\right]}. \tag{17}$$

In a situation where $\Theta = 0$, i.e., all companies can update their prices from period to period, the formula for marginal costs takes the form: $cm_t = \left(\frac{\theta-1}{\theta}\right)\rho_{j,t}$.

2.3. Number of firms

Contemporary empirical research indicates that the process of winding down a business is much less procyclical than the process of starting a business (Broda, Weinstein, 2010; Lee, Mukoyama, 2007). Therefore, modeling changes in the number of operating enterprises based on the assumption of endogeneity of the liquidation process and exogeneity of the company formation process seems to be justified (Bilbiie et al., 2012). It is assumed that there is general equilibrium in the economy, and therefore, in the long run, the number of new firms balances the loss incurred as a result of liquidation. It is assumed that new firms are established when, in the first period of operation, revenues from operations equal the costs incurred. New companies use only labor resources in the production process:

$$f_{j,t} = AH_t l_{0,j,t}, \tag{18}$$

where:

$f_{j,t}$ is the production volume of the new company j,

H_t describes the employment volume as above,

$l_{0,j,t}$ is the volume of labor used in the production process by the new firm j.

Aggregation of (18) leads to:

$$f_t N_{0,t} = AH_t L_{0,t}, \tag{19}$$

where:

$L_{0,t}$ describes the total amount of work devoted to starting a business in period t,

f_t means the production volume of the new company,

f_t also represents the costs of starting a business.

It is assumed that the amount of investment in new companies corresponds to the costs associated with a single-factor production function:

$$I_{N,t} = w_t H_t L_{0,t}. \tag{20}$$

Therefore, relationship (20) can be rewritten using formula (19) as:

$$I_{N,t} = \frac{f_t}{A} w_t N_{0,t}. \tag{21}$$

The total value of new firms $v_t N_{0,t}$ is equal to the amount of investment in new firms $I_{N,t}$. Therefore, the following occurs:

$$v_t = \frac{f_t}{A} w_t. \tag{22}$$

The number of enterprises evolves according to the principle: $N_t = (1 - \delta_N)N_{t-1} + N_{0,t}$.

2.4. Households

The utility function

The utility function of households depends on consumption and the amount of free time. Public consumption was introduced as in Aschauer (1985) and McGrattan (1994). It is assumed that the utility of consumption of an individual household is a linear combination of private consumption $C_{P,t}$ and public consumption $C_{G,t}$:

$$C_t^P = C_{P,t} + \pi C_{G,t}. \tag{23}$$

The parameter π denotes the constant elasticity of substitution between available types of consumption.

Consumers are guided in their choices by consumption habits. Campbell and Cochrane (1999) pointed out that consumption habits naturally result from human psychological conditions. Considering consumption habits in the study allows for better modeling of the sensitivity of consumption as a result of changes in the level of income, and also allows to increase the resistance of the production level to changes and the negative correlation between rates of return and the future level of production (Boldin et al., 2001). The model assumes that the household utility function depends on the difference between current consumption and that from the previous period (i.e., $C_t^P - \phi C_{t-1}^P$, where ϕ is a constant reflecting the strength of consumption habits, $0 \leq \phi \leq 1$).

The utility function for households also takes free time O_t as an argument. Its functional form is as follows:

$$U_t(C_t^P - \phi C_{t-1}^P, O_t) = \gamma \ln(C_t^P - \phi C_{t-1}^P) + (1-\gamma)\ln(O_t), \tag{24}$$

where $0 \leq \gamma \leq 1$ denotes the weights assigned to consumption and free time. The household must decide how to divide the available time between free time O_t and work time L_t:

$$O_t + L_t = 1 \;\Rightarrow\; O_t = 1 - L_t. \tag{25}$$

Households maximize the value stream of expected utility functions. Therefore, what is maximized is the sum of utilities at the current moment and the currently expected future values of the utility function, which are appropriately discounted using the parameter $0 \leq \beta \leq 1$:

$$\mathrm{E_t}\left\{\sum_{t=0}^{\infty} \beta^t U_t(C_t^P - \phi C_{t-1}^P, O_t)\right\}. \tag{26}$$

Budget

The household budget is balanced, income equals expenses. Each household receives remuneration w_t for work L_t depending on the level of employment H_t (less the amount of tax $\tau_{L,t}$), in addition, interest r_t on the physical capital K_t (also taxed $\tau_{K,t}$), it also has a portfolio x_t shares in companies whose value is marked as v_t and which pay dividends from the profits d_t. This money is spent on consumption $C_{P,t}$ (also taxed $\tau_{C,t}$), on capital investments $I_{K,t}$, and reinvests some of the funds in companies, both existing and new ones. By updating their portfolio in the next period, households hope to receive a dividend in the future. The budget constraint for households takes the form:

$$(1+\tau_C)C_{P,t} + I_{K,t} + v_t\left(N_t + N_{0,t}\right)x_{t+1} \\ = (1-\tau_L)w_t H_t L_t + (1-\tau_K)r_t K_t + (v_t + d_t)N_t x_t. \tag{27}$$

Physical capital evolves according to the following principle:

$$K_t = (1-\delta_K)K_{t-1} + I_{K,t}, \tag{28}$$

where $0 < \delta_K < 1$ means the coefficient of depreciation of physical capital, and $I_{K,t}$ means net investment in capital.

Households decide what part of the funds to allocate to capital investments and how much to invest in companies. By investing in companies, households also optimize their portfolio of stocks. The Lagrange problem that households solve comes down to:

$$\max_{\substack{C_{P,t},L_t,\\K_t,x_{t+1}}} \mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \begin{matrix} \gamma\, log\left(C_{P,t} + \pi C_{G,t} - \phi\left(C_{P,t-1} + \pi C_{G,t-1}\right)\right) + (1-\gamma)\, log(1-L_t) \\ -\lambda_{2,t} \left[\begin{matrix} (1+\tau_C)C_{P,t} + K_t - K_{t-1} + v_t\left(N_t + N_{0,t}\right)x_{t+1} - \\ -(1-\tau_L)w_t H_t L_t - (1-\tau_K)(r_t - \delta_K)K_{t-1} - (v_t + d_t)N_t x_t \end{matrix} \right] \end{matrix} \right\}.$$

The first-order conditions for households are the following:

$$\frac{\partial \mathcal{L}}{\partial C_{P,t}}: \quad \beta^t \left[\frac{\gamma}{C_t^P - \phi C_{t-1}^P} - \lambda_{2,t}(1+\tau_C)\right] - \beta^{t+1}\left[\frac{\gamma\phi}{C_{t+1}^P - \phi C_t^P}\right] = 0, \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial L_t}: \quad -(1-\gamma)\frac{1}{1-L_t} + \lambda_{2,t}(1-\tau_L)w_t H_t = 0, \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial K_t}: \quad \beta^t \lambda_{2,t}[(1-\tau_K)(r_t - \delta_K) + 1] - \beta^{t-1}\lambda_{2,t-1} = 0, \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial x_{t+1}}: \quad -\beta^t \lambda_{2,t} v_t\left(N_t + N_{0,t}\right) + \beta^{t+1}\lambda_{2,t+1}(v_{t+1} + d_{t+1})N_{t+1} = 0. \tag{32}$$

2.5. Labor market

The total percentage of employees is H_t. Unemployed people who find a job are immediately employed. The fraction of unemployed relative to the total at time t is equal to:

$$u_t = 1 - H_t. \tag{33}$$

Employment occurs as a result of a meeting between the company and the job seeker (Obstbaum et al., 2023). Companies announce s_t vacancies for which they are looking for employees. The number of new jobs depends on the vacancies and the level of unemployment. The linking function describes the contacts established by interested parties leading to the creation of new positions; it has the form:

$$m_t = \Gamma_t {u_t}^{\sigma} {s_t}^{1-\sigma}, \tag{34}$$

It is an increasing function with respect to both arguments and is continuous and homogeneous of degree one. Its basic feature is captured by the inequality $m_t \le \min[u_t, s_t]$. This corresponds to a situation where at least some job seekers fail to find employment and similarly some vacancies fail to be filled. Due to this, the model allows for unemployment to occur in an economy in equilibrium. Quotient:

$$q_t = \frac{u_t}{s_t}, \tag{35}$$

refers to the number of job seekers per one vacancy. The probability that the company will find an employee to fill the vacant position:

$$q_{1,t} = \frac{m_t}{s_t}. \tag{36}$$

Similarly, the probability that an unemployed person will find a job is:

$$q_{2,t} = \frac{m_t}{u_t}. \tag{37}$$

The average period of employment is $\frac{1}{q_{1,t}}$ and the average time without employment is $\frac{1}{q_{2,t}}$. Furthermore, $\frac{q_{1,t}}{q_{2,t}} = \frac{u_t}{s_t}$, which leads to the conclusion that each additional person looking for a job reduces the chance of finding a job for the unemployed and at the same time increases the chance of the company finding an employee. Similarly, each subsequent vacancy increases the unemployed person's chance of employment and reduces the company's chance of finding an employee.

Exogenous shocks lead to immediate reductions in employment. It is assumed that in period $t-1$ the fraction $\frac{N_t}{N_{t-1}+N_{0,t-1}}$ of companies ceases to exist, moreover, each of the companies that survive until period t dismisses a constant fraction δ_H current employees. Employees who have lost their job start looking for a new job in the next period. Employment results from the need to fill the places left by dismissed employees. The model assumes that people already employed are not looking for work. A company that does not have vacancies is not looking for employees. The equation describing the dynamics of changes in the employment level has the form:

$$H_t = (1-\delta_H)\left(1 - \frac{N_t}{N_{t-1} + N_{0,t-1}}\right)H_{t-1} + m_t.$$

Let λ_t denote the fraction of employees who lose their jobs at period t, then:

$$\lambda_t = \frac{(1-\delta_H)N_t}{N_{t-1} + N_{0,t-1}} + \delta_H, \tag{38}$$

The equation describing changes in the employment level can therefore be written as:

$$H_t = (1-\lambda_t)H_{t-1} + m_t. \tag{39}$$

In equilibrium, the number of people who lose their jobs in each period is equal to the number of people employed during that time. This leads to the relationship:$\lambda_t H_t = q_{2,t} u_t$.

Let J_t and $V_t$denotes, respectively: the discounted value of the filled and unfilled job position at time t. The value of the vacancy is expressed using the following relationship:

$$V_t = -cost_t + (1-\delta_N)\beta\left[q_{1,t+1}J_{t+1} + \left(1-q_{1,t+1}\right)V_{t+1}\right], \tag{40}$$

where $cost_t$ is the cost associated with the process of searching for an employee, $(1-\delta_N)$ is the chance that the company will survive until the next period, β is the discounting factor, $q_{1,t+1}$

is the probability of finding an employee for a vacant job position, $(1-\delta_N)\beta\big[q_{1,t+1}J_{t+1} + \big(1-q_{1,t+1}\big)V_{t+1}\big]$ is the discounted future value of the vacancy.

Similarly, the value of a filled job position is expressed by:

$$J_t = \sigma_{prof}w_t + (1-\delta_N)\beta[(1-\delta_H)J_{t+1} + \delta_H V_{t+1}], \tag{41}$$

where $\sigma_{prof}w_t$ is the difference between the profit brought by the employee and his remuneration, this value in the model depends on the amount of remuneration, $(1-\delta_N)\beta[(1-\delta_H)J_{t+1} + \delta_H V_{t+1}]$ is the discounted future value of the currently filled job position.

Let U_t and W_t denote, respectively: the discounted value of the profits of the unemployed and employed employee at time t. The profits from being unemployed are expressed by the following relationship:

$$U_t = \sigma_{zas}w_t + \beta\big[q_{2,t+1}W_{t+1} + \big(1-q_{2,t+1}\big)U_{t+1}\big], \tag{42}$$

where $\sigma_{zas}w_t$ is the amount of the benefit to which the unemployed person is entitled, $q_{2,t+1}$is the probability of finding a job for the unemployed person in the next period, $\beta\big[q_{2,t+1}W_{t+1} + \big(1-q_{2,t+1}\big)U_{t+1}\big]$ is the future discounted value of profits for the unemployed person.

Similarly, the value of an employed person's profits is expressed by the following relationship:

$$W_t = w_t + \beta[(1-\lambda_{t+1})W_{t+1} + \lambda_{t+1}U_{t+1}], \tag{43}$$

where w_t is the amount of earnings, the remuneration due to the employee, $\lambda_{t+1}$is the fraction of employees who will lose their job at time $t+1$, $\beta[(1-\lambda_{t+1})W_{t+1} + \lambda_{t+1}U_{t+1}]$ is the discounted future value of the employee's earnings.

The firm's net profit from employing an employee is $J_t - V_t$, while the net profit from employment for the employee is $W_t - U_t$. Maximizing the Nash function comes down to determining the amount of remuneration that solves the following.

$$w_t = \arg\max\ (W_t - U_t)^{\sigma_{Nash}}(J_t - V_t)^{1-\sigma_{Nash}}.$$

The value of the σ_{Nash} parameter implies the strength of the employee's bargaining position in relation to the company. The first-order condition that solves the Nash problem takes the form:

$$\sigma_{Nash}(J_t - V_t) = (1-\sigma_{Nash})(W_t - U_t). \tag{44}$$

2.6. Government and central bank

The government's role is reduced to tax collection and income redistribution. The government's consumption preferences are the same as those of households. Moreover, he balances his expenses in each period. The government obtains resources by taxing consumption and income derived from labor and capital. The effective tax rates are denoted by $\tau_{C,t}, \tau_{L,t}$, and $\tau_{K,t}$, respectively. The government budget is as follows:

$$C_{G,t} = \tau_{C,t}C_t + \tau_{L,t}w_tL_tH_t + \tau_{K,t}(r_t - \delta_K)K_t. \tag{45}$$

The central bank's role in the model is to control interest rates. Using available monetary policy instruments, the central bank strives to keep the economy in equilibrium.

The Taylor's rule (1993) relates short-term nominal interest rates to deviations from the equilibrium position for inflation and GDP. The behavior of the central bank in this model is described by the smoothed Taylor rule. According to the rule, the central bank systematically adjusts interest rates in each period, preferring interventions spread over time rather than one-off, strong interventions in the level of interest rates. The linearized equation takes the form:

$$\hat{r}_t = \Psi_0\hat{r}_{t-1} + (1 - \Psi_0)\left[\Psi_1\left(\widehat{\Pi}_t - \overline{\Pi}\right) + \Psi_2\left(\hat{Y}_t - \bar{Y}\right)\right], \tag{46}$$

where $\hat{r}_t$ is the percentage increase in capital return rates r_t, $\hat{Y}_t$ is the percentage increase in GDP Y_t, $\widehat{\Pi}_t$ is the percentage increase in inflation Π_t, $\overline{\Pi}$, $\bar{Y}$ are the values of variables achieved when the economy is in equilibrium, Ψ_0, Ψ_1,Ψ_2 are the structural parameters of the model.

2.7. Market clearing

Aggregation leads to:

$$C_t + I_{K,t} + v_tN_{0,t} = w_tH_tL_t + r_tK_t + d_tN_t. \tag{47}$$

Total consumption expenditures, together with capital investments and investments in new firms, is equal to income from labor, capital income, and dividends paid by firms to their shareholders.

3. Empirical analysis

In the next stage of the study, the obtained model was linearized and then a total of five structural shocks were added to it. These shocks have been linked to the costs of starting a business, changes in the level of public spending, changes in production technology, central bank monetary policy, and changes in employment rates. The choice of structural shocks for the model results from the dynamics of events that directly influenced the Polish economy in recent years.

3.1. Data

The model presented includes five structural shocks. Therefore, five macroeconomic variables related to Poland will be used to estimate its parameters. These are: the number of new enterprises registered in the REGON register, Gross Domestic Product, the rate of changes in average employment in the enterprise sector, the price index of consumer goods and services, and the average monthly gross salary. Due to limited access to historical data on selected variables, the author decided to work with quarterly data, using data from 2012-2023 in the study, which gave a total of 45 observations for each of the variables mentioned.

The model does not consider seasonal fluctuations, and the assumption of a state of general equilibrium results in the need to neutralize any empirically observable trend. Moreover, it should be remembered that after the log-linearization stage, variables interpreted as deviation from the equilibrium state remain in the model. To remove the seasonal component from the data, the author used X-12-ARIMA filtration, a procedure created and widely used, among others, by the US Census Bureau.

3.2. Calibration

The model includes 24 parameters. From this group, 10 parameters are calibrated, and the values obtained for the remaining 14 are the result of Bayesian estimation carried out on real data. The selection of calibrated parameter values involved assigning values that were used by other authors of DSGE models or resulted from the analysis of actual data provided by the Central Statistical Office (Table 1).

Table 1.
Calibrated parameters

Parameter	Value	Comments
β	0.995	Equivalent to a 2% loss of current utility after one year.
δ_N	0.066	The quotient of the number of new and existing non-financial enterprises in Poland from 2015-2023.
δ_H	0.025	Experimentally verified.
τ_C	0.190	Effective VAT rate.
τ_L	0.272	At the personal income tax rate of 18% and tax-deductible costs of 20%.
τ_K	0.190	Tax on capital
$\bar{u}$	0.468	Difference between unity and total employment rate for Poland in 2017.
σ	0.500	Based on literature.
σ_{ZAS}	0.300	Based on literature.
σ_{Nash}	0.500	Based on literature.

Source: Own study based on Baranowski et al. (2013), Torres, (2013); Levis, Polly (2012), Grabek et al. (2010) and Christiano et al. (2005).

3.3. Estimation

The model parameters were obtained using Bayesian estimation. Knowledge about the parameters then comes partly from empirical data and partly from the researcher's subjective assessment. An arbitrarily determined distribution of a parameter is called the a priori distribution. In practice, the parameter estimation process is based on the confrontation of the established a priori distribution with the data. As a result, this leads to a posteriori distribution.

The sample from a posteriori distribution is obtained by using the iterative Metropolis-Hastings algorithm. Five Markov chains were used to estimate marginal distributions using Monte Carlo methods, each of which consisted of one million replications, for which the rejection threshold was set at 25%. Expected values for prior distributions were taken from other DSGE models (Baranowski et al., 2013; Torres, 2013; Levis, Polly, 2012; Grabek et al., 2010; Christiano et al., 2005).

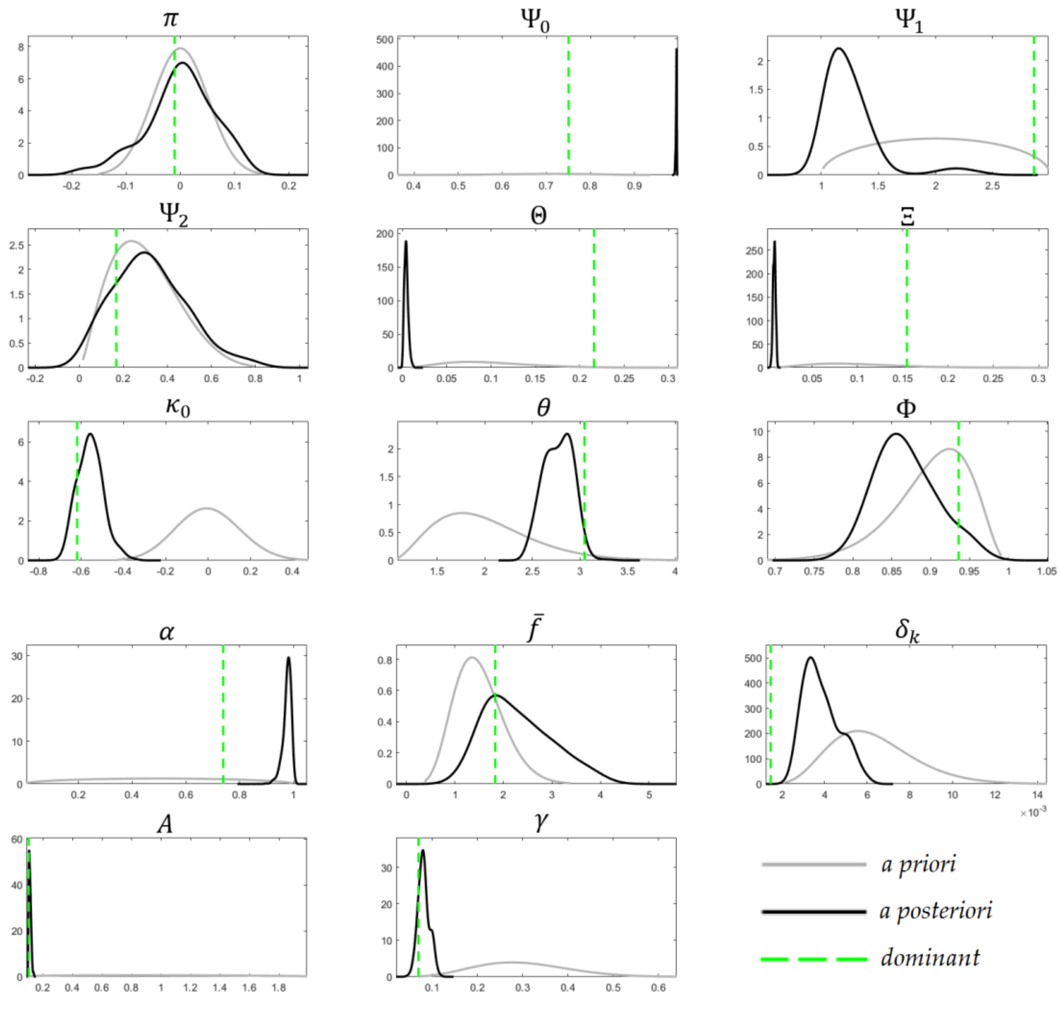


Figure 1. Results of estimating the parameters of the DSGE model using Bayesian techniques.
Source: Own calculations.

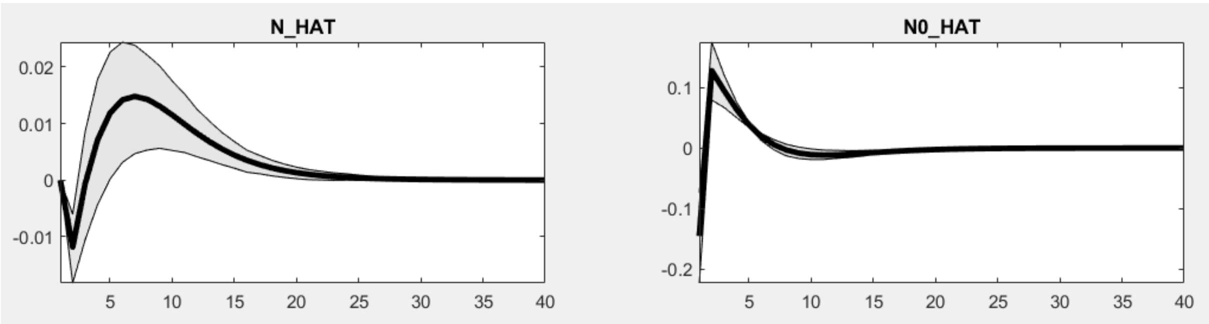
When analyzing the a priori and a posteriori distribution graphs (Figure 1), significant differences are clearly noticeable in all cases except π and Ψ_2. Bayesian estimation in both cases led to a slight change in the expected value of the parameters but did not help reduce the subjectively determined variance of the distributions. In the case of the remaining twelve parameters, the differences concern both the estimated measure of the center and the measure of dispersion, which can be treated as evidence of a good fit of the model to the real data.

4. Results

Impulse response functions (IRF) allow us to answer the question of how the trajectories of deviations from the equilibrium state of model variables develop in response to structural shocks in the model. The assumption of the correct interpretation of the obtained results is that the economy is in a state of equilibrium when the disturbance occurs, and after the disturbance no other disturbances occur. The response of the business population to structural shocks in the model was analyzed. The reactions presented to the impulse concern only new companies and the total number of companies. The window length is 10 years (40 quarters).

4.1. Entry cost

The negative productivity shock regarding new firms increases the costs of doing business. It does not affect the productivity of existing companies, but it reduces the attractiveness of investing in new ones. As a result, the total number of companies is decreasing (Figure 2). A s a result of reduced financing for new companies, consumption increases. Employment levels are falling because fewer new jobs are being created. As a result of the increase in consumption, each new company increases its production level. Since relative prices do not adjust immediately, an increase in production results in an increase in corporate profits. Therefore, this shock generates a negative production gap, which results in the relaxation of the central bank's monetary policy.



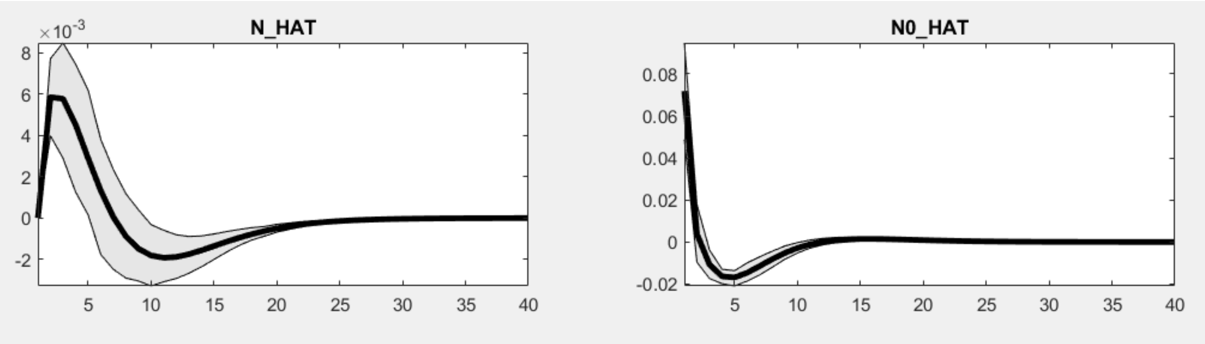
Note: The 95% confidence interval is marked in gray.

Figure 2. Percentage deviations from steady state for the total number (N) and number of new firms (N0) in response to the increase in entry cost.

Source: Own calculations.

4.2. Public spending

Increasing public consumption reduces the availability of consumer goods for households. As a result, production levels increase. This results in a positive production gap, which in turn affects monetary policy and consequently leads to an increase in interest rates. The level of production of existing companies increases (and, consequently, their profits also increase). This increase is not related to a change in production technology; therefore, the increase in production occurs as a result of increasing employment and the creation of new companies. The increasing demand for the employment of new employees by existing companies results in a reduction in the availability of work for new companies, which again translates into a reduction in their number over time (Figure 3).



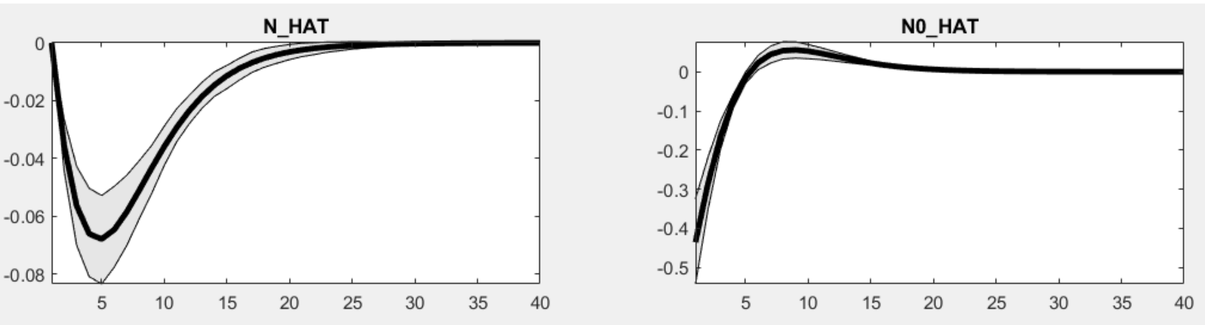
Note: The 95% confidence interval is marked in gray.

Figure 3. Percentage deviations from steady state for the total number (N) and number of new firms (N0) in response to the increase of public spending.

Source: Own calculations.

4.3. Productivity

The increase in productivity resulting from improved production technology will have a direct impact on the price level. As a result, each company will reduce the prices of manufactured goods, which will have a real impact on the level of inflation. The number of companies is determined in advance and, therefore, increasing productivity will increase production. An increase in production combined with a decline in inflation will lead to an increase in the production gap, which will result in a relaxation of monetary policy. An increase in the level of production combined with an increase in the amount of money available will result in an increase in workers' wages. Increasing the level of wages will have a negative impact on the costs of starting a business. This, combined with a decrease in sales prices, will lead to a decrease in the number of new companies (Figure 4).



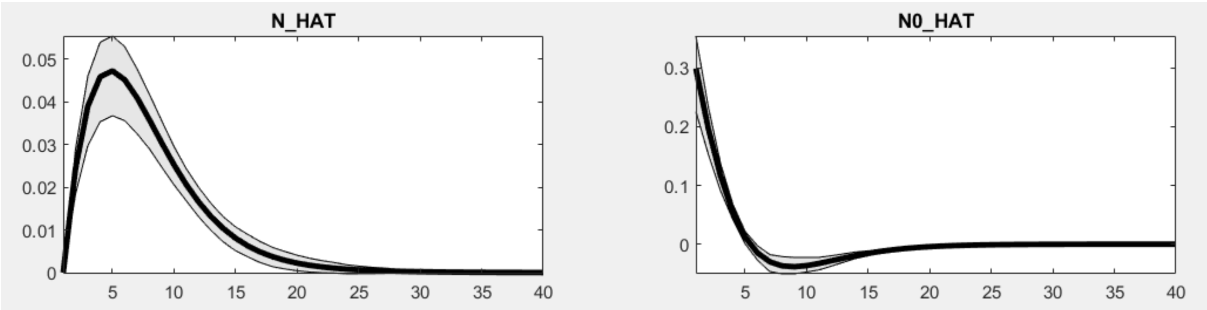
Note: The 95% confidence interval is marked in gray.

Figure 4. Percentage deviations from steady state for the total number (N) and number of new firms (N0) in response to productivity shock.

Source: Own calculations.

4.4. Monetary policy

The tightening of monetary policy causes a decline in the rates of return for capital. This situation leads to increased consumption. As a result, a positive production gap is created. Companies start producing more and therefore have greater profits. New firms are created. To increase production and meet increased consumption, companies increase employment. Increasing employment has a negative impact on the number of new firms over time (Figure 5).



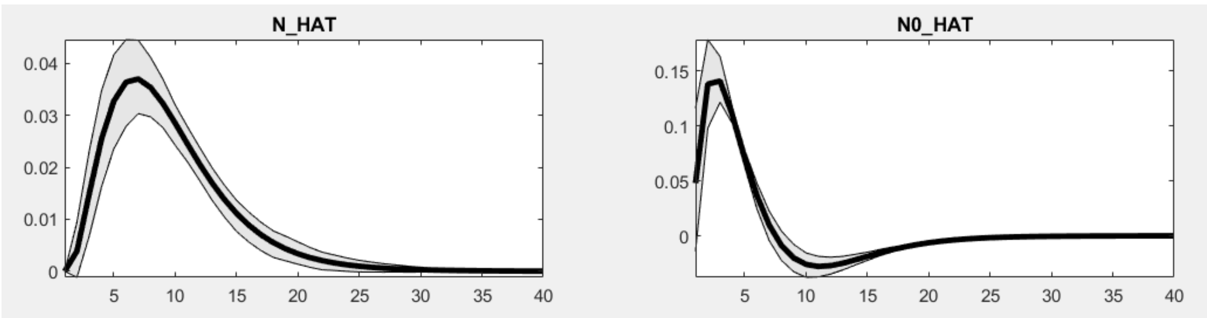
Note: The 95% confidence interval is marked in gray.

Figure 5. Percentage deviations from steady state for the total number (N) and number of new firms (N0) in response to tightening of monetary policy.

Source: Own calculations.

4.5. Employment

The shock associated with reduced employment will negatively impact the level of productivity in the economy. The decline in productivity will have a negative impact on the level of production and, consequently, on the level of consumption. Existing companies will become less productive, and, at the same time, new employees will appear on the labor market. As a result, the costs of starting a new business will decrease. New companies will be established, and the level of production will gradually increase (Figure 6).



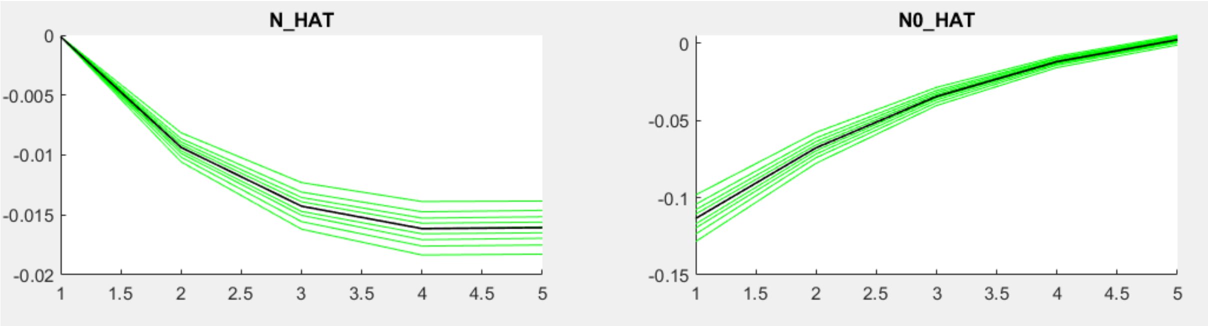
Note: The 95% confidence interval is marked in gray.

Figure 6. Percentage deviations from steady state for the total number (N) and number of new firms (N0) in response to tightening of employment shock.

Source: Own calculations.

4.6. Predictions

It is not easy to estimate the number of companies that operate actively in Poland. REGON statistics ‘swell’ year by year because they do not consider the division into registered and active entities of the national economy. This was the basic reason for not including this variable in the group of observable variables of the model. However, a good idea of the number of companies can be obtained by observing the number of new entries, and the statistics themselves are reliable. Based on these data, the model presented forecast the total number and the number of new firms in Poland (Figure 7). By analyzing the obtained figures, the number of new companies is currently relatively small. A gradual return to balance is expected in five quarters. In the context of the total number of companies, a decline is expected. This is a natural consequence of the fact that there will be fewer new firms.



Note: The black line indicates the forecast in means for the analyzed variables, obtained using the Kalman filter. Green lines represent deciles of the fitted uncertainty distribution.

Figure 7. Forecasts for percentage deviations from steady state for the total number (N) and number of new firms (N0).

Source: Own calculations.

5. Concluding remarks

In conclusion, the application of DSGE class models to study the relationship between macroeconomic variables and the business demography in Poland offers valuable insights into the complex interactions shaping the business landscape. Several key findings emerge from this analysis:

a) Recent Economic Challenges: Recent events, including the COVID-19 pandemic and the war in Ukraine, have presented unique challenges to the Polish economy. The conclusion of this research is that the Polish economy is sensitive to various shocks, including negative productivity shocks, changes in public consumption, improvements in production technology, changes in monetary policy, and employment fluctuations. These shocks have ripple effects on business creation, growth, and contraction.

b) Monetary Policy's Role: The text highlights the significant influence of monetary policy on business dynamics. Changes in interest rates and monetary policy measures have direct implications for both the number of new firms and the behavior of existing companies.
c) Employment's Impact: Employment levels play a crucial role in shaping the entrepreneurial landscape. Reductions in employment costs can stimulate the establishment of new companies, while increased employment by existing firms can limit opportunities for newcomers.
d) Data Challenges: The analysis acknowledges challenges in accurately estimating the total number of active companies in Poland, emphasizing the use of new entry statistics as a proxy. This underscores the importance of improving data accuracy for more precise economic analysis.
e) Policy Implications: The findings have important policy implications. Policymakers should consider the potential consequences of their decisions on the business environment, especially in terms of labor market regulations, public spending, and monetary policy.
f) Forecasting Business Trends: The presented model forecasts a decline in the total number of companies in Poland, reflecting the current economic conditions. This forecast can guide policymakers and businesses in planning for the future.

In sum, the analysis provides a valuable framework for understanding how macroeconomic variables influence the business demography in Poland. It sheds light on the intricate dynamics at play and offers valuable insights for policymakers and businesses seeking to navigate this evolving landscape.

References

1. Aschauer, D. (1985). Fiscal policy and aggregate demand. *American Economics Review, vol.75(1),* 117-127.
2. Baranowski, P., Gałecka-Burdziak, E., Górajski, M., Malaczewski, M., Szafrański, G. (2013). *Inflacja a mechanizmy aktualizacji cen. Studium dla Polski.* Łódź: Wydawnictwo Uniwersytetu Łódzkiego.
3. Bernard, A.B., Redding, S.J., Schott, P.K. (2010). Multiple-Product Firms and Product Switching. *American Economic Review, vol.100(1),* 70-97.
4. Bilbiie, F.O., Ghironi, F., Melitz, M.J. (2012). Endogenous Entry, Product Variety, and Business Cycles. *Journal of Political Economy, vol.120,* 304-345.
5. Blanchard, O.J., Kahn, C.M. (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica, vol.48(5),* 1305-1311.

6. Blanchard, O.J., Perotti, R. (2002). An empirical characterization of the dynamic effects of change in goverment spending and taxes on output. *Quarterly Journal of Economics, vol.117(4),* 1329-1368.
7. Boldin, M., Christiano, L., Fisher, J. (2001). Habit persistence, asset returns, and the business cycle. *Americn Economic Review, vol.91(1*), 149-166.
8. Broda, C., Weinstein, D.E. (2010) .Product Creation and Destruction: Evidence and Price Implications. *American Economic Review, vol.100(3),* 691-723.
9. Calvo, G.A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics, vol. 12*, 383-398.
10. Campbell, J., Cochrane, J. (1999). By force of habit: A consumption-based explanation of aggregate stack market behavior. *Journal of Political Economy, vol.107(2),* 205-251.
11. Casares, M., Khan, H., Poutineau, J-C. (2020). The extensive margin and US aggregate fluctuations: A quantitative assessment. *Journal of Economic Dynamics and Control, vol. 120*, 103997.
12. Christiano, L.J., Eichenbaum, M.S., Evans, C.L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy, vol.113(1),* 1-45
13. Christiano, L.J., Eichenbaum, M.S., Trabandt, M. (2018). On DSGE Models. *Journal of Economic Perspectives, American Economic Association, vol. 32(3),* pp. 113-140.
14. Colciago, A., Etro, F. (2010). Real business cycles with Cournot competition and endogenous entry. *Journal of Macroeconomics, Elsevier, vol. 32(4),* 1101-1117.
15. Davis, S.J., Haltiwanger, J.C., Schuh, S. (2002). *Job creation and destruction*. Massachusetts: MIT Press Books.
16. Dixit, A.K., Stiglitz, J.E. (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review, vol. 67(3),* 297-308.
17. Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.
18. Gali, J., Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics, vol. 44(2),* 195-222.
19. Gali, J., Smets, F., Wouters, R. (2012)., Unemployment in an Estimated New Keynesian Model. *NBER Working Paper, No. 17084*, 1-53.
20. Grabek, G., Kłos, B., Koloch, G. (2010). *SOEPL2009 – Model DSGE małej otwartej gospodarki estymowany na polskich danych. Specyfikacja, oceny parametrów, zastosowania.* Warszawa: Narodowy Bank Polski.
21. Jaravel, X. (2019). The Unequal Gains from Product Innovations: Evidence from the U.S. Retail Sector. *The Quarterly Journal of Economics, 134(2),* 715-783.
22. Lee, Y., Mukoyama, T. (2007). Entry, Exit, and Plant-level Dynamics over the Business Cycle. *Federal Reserve Bank of Cleveland Working Paper, 07-18*.

23. Levis, V., Poilly, C. (2012). Firm Entry, Markups and the Monetary Transmission Mechanism. *Journal of Monetary Economics, vol.59(7),* 670-685.
24. McGrattan, E. (1994). The macroeconomic effects of distortionary taxation. *Journal of Monetary Economics, vol. 33(3),* 573-601.
25. Obstbaum, M., Oinonen, S., Pönkä, H., Vanhala, J., Vilmi, L. (2023). Transmission of recent shocks in a labour - DSGE model with wage rigidity. *BoF Economics Review, vol. 1,* 1-47.
26. Perotti, R. (2008). In search of the transmission mechanism of fiscal policy. *NBER Macroeconomics Annual, vol. 22,* 169-226.
27. Prescott, E., Kydland, F. (1982). Time to Build and Aggregate Fluctuations. *Econometrica, vol. 50,* 1345-1371.
28. Taylor, J.B. (1993). Discretion Versus Policy Rules in Practice. *Carnegie-Rochester Series on Public Policy, vol. 39,* 195-214.
29. Torres, J.L. (2013). *Introduction to Dynamic Macroeconomic General Equilibrium Models. Series in Economic Methodology*. Delaware: Vernon Press.