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TOWARDS A SMART CITY: MODELLING A BIKE-SHARING STATION VIA A QUEUEING LOSS SYSTEM

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Purpose: The article aims to build a mathematical model of a bike-sharing station based on an appropriate queueing system and show the model's usefulness in practice.
Design/methodology/approach: In designing the model, constructing a queueing system described by exponential distributions with a finite accumulating buffer was used. The existence of the steady state of the system and the global balance principle were used to obtain analytical results.
Findings: The most important analytical results are the stationary probability distribution of the number of rented bikes, the so-called loss probability (the probability that the customer has to resign from sharing due to the lack of bikes), as well as the average (mean) values of the number of rented bikes.
Originality/value: The paper fits into the broadly understood trend of research related to the smart city concept. The proposed model may be beneficial in practice when designing specific solutions related to the development of bicycle rental stations.

Keywords: Bike-sharing station, customer loss, queueing system, smart city, stationary state.

Category of the paper: Research paper.

1. Introduction

The concept of smart cities requires an in-depth analysis of the market and consumer needs. This, of course, involves proposing appropriate communication solutions, including bicycle rental stations. Analytical models in this area can, therefore be actively used in practice. The use of queueing models in practical modeling is common today and is constantly gaining in importance (see, e.g., Bose, 2002; Ng, Soong, 2008; Chan, 2014; Shortle et al., 2018 and Lakatos et al., 2019). Queueing systems are used to design network protocols and solve logistics and transport issues (including communication nodes and traffic control systems).

An overview of the results regarding the modeling of bicycle rental stations can be found in (Fishman, 2016). In (Ashqar et al., 2017; Yang et al., 2018), artificial intelligence algorithms, particularly deep machine learning, were used to analyze the functioning of bicycle rental stations. The article (Wang et al., 2015) considered the influence of the location of a bicycle rental station on its operational characteristics. An optimization approach to this issue was proposed in (Qian et al., 2022).

The paper proposes a queueing model describing renting and returning bicycles from a bike-sharing station. This model is based on exponential distributions describing the process of incoming customers (who want to rent a bike) and servicing them (understood here as the time of using the rented bike). The maximum system size corresponds to the number of service stations in an appropriately chosen queueing model. Indeed, by a single service station, we can mean a single bicycle in use. Occupying all service stations, therefore means renting all available bicycles. This number is equal to the maximum number of customers in the system (in the model, we assume that when the bike-sharing station is empty, the potential customer does not wait for a bike but resigns from the station's services).

For the stationary state of the system, after its stabilization, analytical results are presented for the distribution of the number of customers, understood here as the number of rented bicycles. In particular, a representation for the so-called loss probability, i.e., the probability that all bicycles are rented, is given, as well as the formula for the average (mean) value of the number of rented bicycles.

2. Model description

In the article, we analyze a mathematical model of a single bike-sharing station described by means of a finite-capacity queueing system with Poisson arrivals of customers with a given rate a. The Poisson arrival stream describes potential customers of the station who would like to rent a bike. In practice, the arrival rate changes in time: its values can differ in different periods. A typical engineering approach in such a situation is to divide the observation period into a finite number of subperiods in which the intensity of arrivals can be accepted to be constant. The service process reflects the process of using the rented bike: we assume that successive processing times are independent and identically distributed random variables with the mean b^{-1}. The station "capacity" equals $m \geq 1$, i.e., we have m bikes that can be rented. If a customer occurs when there are no bikes available for rent, he leaves the station without service (in the "language" of queueing theory, we say that such a customer is "lost"). Thus, using the classical Kendall notation, the considered queueing model can be classified as the *M/M/m/m*-type system (a kind of system with customer losses without a waiting room).

Critical from the point of view of ensuring the appropriate quality of customer service (QoS) is constant monitoring of the number of bicycles available for rent. Thanks to this, it is possible to optimally use the station - avoiding a situation in which no bikes are available for rent for a long time during the day, or a small number of them is rented compared to the total number of bikes offered. When describing the model probabilistically using an appropriate queueing system, we are interested in the probability distribution of the number of bicycles available at the station, and the probability that an arriving customer will find the station empty (no bicycles to rent) will be significant for us. We will consider the model in the steady state, i.e. at $t \to \infty$. Of course, because the system contains a finite buffer, regardless of the intensity of customer input and the speed of their service, the steady state exists (see e.g. Adan, Resing, 2015; Tijms, 2003; Heyman, Sobel, 1982 for basics of stochastic modeling in this area).

3. Analytical results

Let us introduce the following notation:

$$q_n \stackrel{\text{def}}{=} P\{X = n\}, \tag{1}$$

where X stands for the number of rented bikes, $n \in \{0, 1, \dots, m\}$. Obviously, the following normalization condition is satisfied:

$$\sum_{n=0}^{m} q_n = 1. \tag{2}$$

The global balance principle (see e.g., Adan, Resing, 2015), which can be applied to any steady-state queueing model, states that the output flow of customers from a given state is equal to the input flow to the state. Note that the output stream of customers to state 0 (0 bikes rented, all bikes available) is equal to the product of the intensity of customers coming to the service station and the probability q_0 that the system is in state 0. In other words, the fraction aq_0 of the input stream corresponds to the transition from state 0 to state 1. Similarly, the input stream to state 0 "comes" from state 1 and amounts to bq_1, where b is the intensity of customer service (in our bike-sharing station model, it corresponds to the intensity of returning rented bicycles). Similarly, the fraction bq_1 of the total stream describing the intensity of customer service per time unit corresponds to the transition from state 0 to state 1. Comparing both streams to each other, we obtain the equation

$$aq_0 = bq_1. \tag{3}$$

Now consider state 1. The customer output stream from this corresponds to a transition from state 1 to state 2 (customer influence), but also to a transition from state 1 to state 0 (customer service). Therefore it is equal to $aq_1 + bq_1 = (a + b)q_1$. In turn, the input stream to state 1 is related to the arrival of the customer to the "empty" system (aq_0) or with the end of customer service in a system that was in state 2 ($2bq_2$), it is then equal to $aq_1 + 2bq_2$. The quantity $2b$ is related to the fact that in the considered queueing model "being" in state 2, the intensity of "customer service" is $2b$ – each customer who rented a bike returns it with intensity b (merging property of Poisson process describing the service in the considered model). Equating both streams to each other, we obtain the equation

$$(a+b)q_1 = aq_0 + 2bq_2. \tag{4}$$

For state 2, in consequence, we obtain an analogous equation, applying the global balance principle

$$(a+b)q_2 = aq_1 + 3bq_3. \tag{5}$$

In general, for state n, where $n \in \{1, \dots, m-1\}$, we have the following equation:

$$(a+b)q_n = aq_{n-1} + nbq_n. \tag{6}$$

A specific situation occurs for state m, corresponding to the situation in which all available bicycles have been rented. It is no longer possible to move from this state to a higher state so that the appropriate equilibrium equation will be

$$aq_{m-1} = mbq_m. \tag{7}$$

Let us observe that the consequence of the previous equations is the following recursive formula:

$$aq_{n-1} = nbq_n, \tag{8}$$

where $n \in \{1, \dots, m-1\}$.

The above recurrence can be solved explicitly. The solution, so the representation for the stationary number of rented bikes in the station containing strictly m bikes, has the following form (see also e.g. Adan, Resing, 2015):

$$q_n = \left[\frac{(a/b)^n}{n!}\right] : \left[\sum_{i=0}^{m} \frac{(a/b)^i}{i!}\right] = \frac{\rho^n/n!}{\sum_{i=0}^{m} \rho^i/i!}, \tag{9}$$

where $n \in \{0, 1, \dots, m\}$ and $\rho = \frac{a}{b}$ denote the so-called offered load (traffic load) in the considered queueing model and defines the proportion between the arrival rate and service speed.

The so-called blocking probability is of particular importance for assessing the system is functioning, i.e., the probability that all available bikes will be rented and, consequently, the upcoming customer will be lost. This probability is equal to q_m, and hence

$$q_{block} = \frac{\rho^m/m!}{\sum_{i=0}^{m} \rho^i/i!}. \tag{10}$$

It is possible to use blocking probability to represent the average (mean) number of rented bicycles without calculating the sum of the appropriate numerical series. We have (see e.g. Adan, Resing, 2015)

$$E(X) = \rho(1 - q_{block}). \tag{11}$$

4. Numerical examples

The numerical results were obtained by a Python program using a math library, which code is presented in figure 1 below.

```python
import math

def calculate_probabilities(a, b, m):
    ρ = a / b
    denominator_sum = sum([(ρ ** i) / math.factorial(i) for i in range(m + 1)])
    probabilities = [(ρ ** n) / (math.factorial(n) * denominator_sum) for n in range(m + 1)]
    return probabilities

def calculate_blocking_probability(ρ, m):
    return (ρ ** m) / math.factorial(m)

def calculate_mean_number_of_bikes(ρ, blocking_probability):
    return ρ * (1 - blocking_probability)

if __name__ == "__main__":
    a = 1.5  # Arrival rate
    b = 1.0  # Service rate (return rate)
    m = 5    # Number of available bikes

    ρ = a / b

    probabilities = calculate_probabilities(a, b, m)
    blocking_probability = calculate_blocking_probability(ρ, m)
    mean_bikes = calculate_mean_number_of_bikes(ρ, blocking_probability)

    print("Probabilities q_n:")
    for n, prob in enumerate(probabilities):
        print(f"q_{n}: {prob:.5f}")

    print("\nBlocking Probability (q_block):")
    print(f"q_block: {blocking_probability:.5f}")

    print("\nMean Number of Rented Bicycles (E(X)):")
    print(f"E(X): {mean_bikes:.5f}")
```

Figure 1. Program code.

Source: Authors' own.

The results differ depending on the input parameters (a, b, and m). The numerical results are presented in Table 1.

Table 1.
Numerical results

Input parameters: a = 1.5, b = 1.0, m = 5	Input parameters: a = 2, b = 1.0, m = 5	Input parameters: a = 1.5, b = 1.5, m = 5	Input parameters: a = 1.5, b = 2.0, m = 5
Probabilities q_n: q_0: 0.22413 q_1: 0.33619 q_2: 0.25214 q_3: 0.12607 q_4: 0.04728 q_5: 0.01418	Probabilities q_n: q_0: 0.13761 q_1: 0.27523 q_2: 0.27523 q_3: 0.18349 q_4: 0.09174 q_5: 0.03670	Probabilities q_n: q_0: 0.36810 q_1: 0.36810 q_2: 0.18405 q_3: 0.06135 q_4: 0.01534 q_5: 0.00307	Probabilities q_n: q_0: 0.47243 q_1: 0.35432 q_2: 0.13287 q_3: 0.03322 q_4: 0.00623 q_5: 0.00093
Blocking probability (q_block): q_block: 0.06328	Blocking probability (q_block): q_block: 0.26667	Blocking probability (q_block): q_block: 0.00833	Blocking probability (q_block): q_block: 0.00198
Mean Number of Rente d Bicycles (E(X)): E(X): 1.40508	Mean Number of Rented Bicycles (E(X)): E(X): 1.46667	Mean Number of Rented Bicycles (E(X)): E(X): 0.99167	Mean Number of Rented Bicycles (E(X)): E(X): 0.74852

5. Source: Authors' own

The results show that if the arrival rate increases, the mean of rented bicycles also increases. In case the service rate increases, the mean of rented bicycles decreases. The numerical examples show that the model works properly and can be easily described from a 'human's point of view'.

6. Conclusions

To declare the city as smart, it is necessary to provide different services for the inhabitants. One of the most popular services is a bike-sharing station. As we assume, the number of users, the number of bikes, and many more parameters should be considered as crucial to the proper service of inhabitants. The presented paper shows that the queuing model can provide the proper satisfaction level based on real data for a chosen city. The satisfaction of citizens is one of the requested values for a smart city.

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