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CRITICAL ANALYSIS OF CLASSICAL SCENARIO-BASED DECISION RULES FOR PURE STRATEGY SEARCHING

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Purpose: The contribution briefly presents the essence and applications of the well-known classical decision rules designed for decision making under uncertainty with unknown probabilities and based on scenario planning. We concentrate on games against nature, pure strategy searching and one-criterion optimization.
Design/methodology/approach: The main goal of this work is to analyse numerous case studies and formulate conclusions concerning the properties (in particular drawbacks and limitations) of the aforementioned strategic procedures.
Findings: The paper focuses on the limited usefulness of classical decision rules in real economic decision problems. It is advised to apply them very carefully.
Research limitations/implications: A similar analysis for mixed strategy searching and multi-criteria optimization should be performed within a future research.
Originality/value: The work offers numerous practical guidelines modifying existing procedures and allowing decision makers to obtain logic recommendations.

Keywords: classical uncertainty one-criterion decision rules, scenario planning, pure strategies, decision maker's preferences, economic case studies

Category of the paper: Case study, research paper.

1. Introduction

In the contribution we deeply explore the classical decision rules (CDR) based on scenario planning (SP) and designed for: (1) decision making under uncertainty with unknown probabilities, (2) games against nature, (3) one-criterion optimization and (4) pure strategy searching. Within the framework of scenario planning the decision problem is defined in the form of a set of possible decisions $D = \{D_1, \ldots, D_j, \ldots, D_n\}$, a set of possible scenarios $S = \{S_1, \ldots, S_i, \ldots, S_m\}$ and a payoff matrix containing $m \times n$ outcomes denoted by a_{ij} (this symbol signifies the performance of a given criterion by decision D_j provided that scenario S_i happens). Payoff matrices representing the aforementioned problems can be replaced by decision trees, see e.g. the software SilverDecisions (Kamiński, Jakubczyk, and Szufel, 2017). A pure strategy, as opposed to a mixed strategy, allows the decision maker (DM) to select and perform only one accessible alternative (Gaspars-Wieloch, 2018b). In the work we only investigate the case where particular decisions depend on the same set of scenarios.

We intend to verify the usefulness of six classical strategic procedures, that is decision rules handling uncertainty with unknown probabilities: Wald rule, max-max rule, Hurwicz rule, Savage rule, max-min joy criterion and Bayes rule. That is why approaches devoted to uncertainty with known probabilities, like max-min expected utility, reliability-weighted expected utility, Ellsberg’s index, Gärdenfors's and Sahlin's modified MMEU and Levi's lexicographical test (Hansson, 2005; 2011), are not the subject of this article.

CDR have mainly been formulated in the twentieth century (e.g. Wald rule, Hurwicz rule), but the oldest one even dates back to the eighteenth century (Bayes rule). They are well-known among researchers and partially applied by practitioners (Clarke, 2008). The literature describing these approaches is extremely rich (e.g. Chateauneuf, Cohen, and Jaffray, 2006; Czerwiński, 1972; Gaspars-Wieloch, 2018a; 2018b; Hansson, 2005; 2011, Ignasiak, 1996; Kaufmann, and Faure, 1965; Sikora, 2008; Trzaskalik, 2008). It is obvious that CDR are not flawless (Hansson, 2011; Milnor, 1954; Officer, and Anderson, 1968), but in this contribution we concentrate on the drawbacks which have not been previously diagnosed.

The paper is organized as follows. Section 2 briefly describes the essence and applications of CDR. Section 3 presents diverse economic case studies solved by means of classical decision rules. Section 4 discusses the defects and limitations of the aforementioned procedures. Conclusions are gathered in Section 5.

2. Classical decision rules – description

We explain below the idea of the classical decision rules with the assumption that the target of the solved problems is maximized.

The **Wald rule** (max-min rule) consists in defining the security level (the worst possible outcome, w_j) for each alternative and choosing the option with the maximal security level (Wald, 1950). This rule represents extreme prudence.

The **max-max rule** requires finding the hope level (the best outcome, m_j) for each option and selecting the one with the maximal hope level.

Within the **Hurwicz rule** (optimism-pessimism index, Hurwicz, 1952) the DM is supposed to estimate their pessimism coefficient α from the interval [0,1]. It is equal to 1 for extreme pessimists (expecting the occurrence of scenarios with the worst outcomes) and 0 for extreme optimists. Then, for each option, a weighted average is computed according to equation (1) and the alternative with the highest index is treated as the best.

$$h_j = \alpha \cdot w_j + (1 - \alpha) \cdot m_j \tag{1}$$

The **Savage rule** (min-max regret criterion, Savage, 1951) advises the DM to (1) produce a regret matrix containing differences between the maximal outcome gained within a given scenario and the outcome itself, and (2) select the option with the lowest maximal regret.

The **max-min joy criterion** (Hayashi, 2008) suggests generating a relative profit matrix containing differences between the minimal payoff gained within a given scenario and the outcome itself, and (2) choosing the variant with the highest minimal relative profit.

The **Bayes rule** (also known as Bernoulli rule, Laplace rule or principle of insufficient reason) refers to the statement that equal probabilities can be assigned to scenarios if the DM has no reason to believe that one of them is more likely to occur than another. This approach recommends the alternative with the highest arithmetical average of outcomes.

When comparing the construction and applications of the procedures presented above, we can draw the following conclusions (Gaspars-Wieloch, 2018a; 2018b):

1) The Wald, Savage, Hayashi and max-max rules take only one scenario for each alternative into account while the Hurwicz rule considers two scenarios and the Bayes rule – all the available scenarios.
2) The Wald, Savage and Hayashi rules are only devoted to extreme pessimists and the max-max rule is merely designed for radical optimists, but the Hurwicz rule seems to be more practicable since it can be applied by diverse types of decisions makers.
3) Although the Wald, Savage and Hayashi rules have similar applications, they can make different recommendations. The constructions of the Savage and Hayashi procedures are very close to each other, but again – their solutions may be different.
4) The first five approaches find application in one-shot decision problems (the selected variant can be performed only once). The last approach is for multi-shot decisions.
5) In the Savage and Hayashi rules the payoff matrix structure influences the final solution. For the remaining approaches the ranking of decisions does not change after modifying the sequence of the outcomes connected with particular options.

3. Economic case studies and results

In this section we present and solve five economic decision problems by means of the procedures described in the previous section. In each case we assume that the degree of novelty of the analysed problems is so high (due to their innovation or innovative character) that the probability estimation is impossible (lack of sufficient knowledge and historical data). The Reader can find the comments concerning the obtained recommendations in the next section.

CASE 1. The case refers to Net Present Value (NPV) project estimation (Gaspars-Wieloch, 2019). Investors A and B intend to analyse 3 independent sets of projects: $P_1 = \{D_1, D_2, D_3\}$, $P_2 = \{D_4, D_5, D_6\}$, $P_3 = \{D_7, D_8, D_9\}$ on the basis of NPV (i.e. there are three independent decision problems). Investor A is an extreme pessimist while investor B is a radical optimist. Table 1 shows NPVs estimated by experts and the Wald (for investor A) and max-max (for investor B) indices. According to them investor A should choose projects D_1, D_6 and D_7 (but D_8 and D_9 are just as good as D_7). Investor B ought to select projects D_1 (but D_2 and D_3 are just as good as D_1), D_4 and D_7 (but D_8 and D_9 are just as good as D_7).

Table 1
Case 1. Net Present Values (in thousands Euros) and indices

Scenarios	Set of projects P_1			Set of projects P_2			Set of projects P_3		
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9
S_1	600	-100	10	450	300	2	5	5	3,000
S_2	600	-500	20	490	200	3	5	3,000	5
S_3	600	600	50	480	1	3	5	5	3,000
S_4	600	-50	30	0	100	4	5	3,000	3,000
S_5	580	-100	600	500	150	3	3,000	5	3,000
Wald index w_j	<u>580</u>	-500	10	0	1	<u>2</u>	<u>5</u>	<u>5</u>	<u>5</u>
Max-max index m_j	<u>600</u>	<u>600</u>	<u>600</u>	<u>500</u>	300	4	<u>3,000</u>	<u>3,000</u>	<u>3,000</u>

Source: Prepared by the author.

CASE 2. The case is connected with the spare parts quantity problem. Investor C, who is an extreme optimist (hence, let's use the max-max rule), wants to buy an appropriate number of spare parts with the purchase of a machine. Thus, they must: I. estimate the unit purchase cost of the subassembly together with the purchase of the whole device (e.g. $c_1 = 5$), the unit purchase cost just after the failure (e.g. $c_2 = 12$), II. calculate two types of losses: (1) $s_1 = c_1 = 5$ denoting the unit loss due to the excess of spare parts, and (2) $s_2 = c_2 - c_1 = 7$ signifying the unit loss due to the shortage of spare parts, and III. choose the sets of possible scenarios (demands for spare parts) and decisions (order quantities), e.g. 1, 2, 3 and 4 units. These data allow them to generate a payoff matrix (Table 2). According to the max-max indices all the strategies are equivalent.

Table 2
Case 2. Payoff matrix for the spare parts quantity problem (in hundreds dollars) and indices

Scenarios (demand for spare parts)	Decisions (order quantities)			
	1	2	3	4
1	0	-5	-10	-15
2	-7	0	-5	-10
3	-14	-7	0	-5
4	-21	-14	-7	0
Max-max index m_j	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>

Source: Prepared by the author.

CASE 3. Investors D, E, F and G are interested in buying securities (one type). Each investor represents a different type of DM: α_D = 0.8, α_E = 0.6, α_F = 0.4, α_G = 0.2. Table 3 contains estimated monthly rates of return connected with particular securities and the Hurwicz indices. We conclude that, independently of the pessimism level, investors should buy securities D_1.

Table 3

Case 3. Monthly rates of return (in %) and Hurwicz indices

Scenarios	**Decisions (securities)**			
	D_1	**D_2**	**D_3**	**D_4**
S_1	-0.3	6.5	6.6	-1
S_2	-0.4	6.6	-1	6.5
S_3	-0.9	-1	0	6.3
S_4	-0.5	6.6	-0.5	6.4
S_5	6.6	6.5	0.2	6.2
Hurwicz index h_j (α_D=0.8)	0.6	0.52	0.52	0.5
Hurwicz index hj (α_E=0.6)	2.1	2.04	2.04	2.0
Hurwicz index hj (α_F=0.4)	3.6	3.56	3.56	3.5
Hurwicz index hj (α_G=0.2)	5.1	5.08	5.08	5.0

Source: Prepared by the author.

CASE 4. This case is related to the so-called newsvendor problem (Gaspars-Wieloch, 2017). Eleven retailers (each with a different attitude towards risk: α_A = 0.0, α_B = 0.1, α_C = 0.2, …, α_J = 0.9, α_K = 1.0) independently start selling the same innovative product without keeping inventory. They intend to buy a proper quantity of that good at the beginning of a period in order to satisfy the demand in this period. Hence, they have to: I. estimate the unit purchase cost of the product (e.g. c_1 = 500), the selling price of this product (e.g. c_2 = 800) and the price of leftover items (e.g. c_3 = 200), II. calculate the unit gain $g = c_2 - c_1$ = 300 and the unit loss $l = c_1 - c_3$ = 300, and III. choose the sets of possible scenarios (demands) and decisions (supplies), e.g. 1, 2, 3 and 4 units per period. These data allow generating a payoff matrix (Table 4) with the Hurwicz indices. Results indicating the best solutions have been underlined.

Table 4

Case 4. Payoff matrix for the newsvendor problem (in Euros) and Hurwicz indices

Scenarios (demands)	**Decisions (supplies)**			
	1	**2**	**3**	**4**
1	300	0	-300	-600
2	300	600	300	0
3	300	600	900	600
4	300	600	900	1,200
Hurwicz index h_j (α_A=0.0)	300	600	900	<u>1,200</u>
Hurwicz index h_j (α_B=0.1)	300	540	780	<u>1,020</u>
Hurwicz index h_j (α_C=0.2)	300	480	660	<u>840</u>
Hurwicz index h_j (α_D=0.3)	300	420	540	<u>660</u>
Hurwicz index h_j (α_E=0.4)	300	360	420	<u>480</u>
Hurwicz index h_j (α_F=0.5)	<u>300</u>	<u>300</u>	<u>300</u>	<u>300</u>
Hurwicz index h_j (α_G=0.6)	<u>300</u>	240	180	120
Hurwicz index h_j (α_H=0.7)	<u>300</u>	180	60	-60
Hurwicz index hj (α_I=0.8)	<u>300</u>	120	-60	-240
Hurwicz index hj (α_J=0.9)	<u>300</u>	60	-180	-420
Hurwicz index hj (α_K=1.0)	<u>300</u>	0	-300	-600

Source: Prepared by the author.

CASE 5. The case concerns project selection on the basis annuals profits. Investor H is a pessimist, but additionally he would like to find the best strategy by comparing its profits with outcomes connected with the remaining strategies within particular scenarios. This entails the necessity to apply the Savage or Hayashi rules which enable one to take the position of a given payoff into account. Table 5 presents the initial profits, relative losses, relative profits as well as the Savage and Hayashi indices. Project D_1 is the best according to the Savage rule. For the Hayashi approach all the projects are equivalent.

Table 5
Case 5. Project annual profits, rel. losses, rel. profits (*in thousands dollars*) *and indices*

Scenarios	**Decisions (projects)**			
	D_1	**D_2**	**D_3**	**D_4**
S_1	200	68	-180	170
S_2	100	-55	39	37
S_3	150	42	140	-100
S_4	40	79	45	120
Scenarios	**Relative losses**			
S_1	0	132	380	30
S_2	0	155	61	63
S_3	0	108	10	250
S_4	80	41	75	0
Savage index s_j	<u>80</u>	155	380	250
Scenarios	**Relative profits**			
S_1	380	248	0	350
S_2	155	0	94	92
S_3	250	142	240	0
S_4	0	39	5	80
Hayashi index ha_j	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>

Source: Prepared by the author.

4. Discussion

In this section we discuss the solutions generated by particular decision rules and analyse the drawbacks, limitations and usefulness of the aforementioned approaches. The emphasize is put on issues which have not been described by other researchers.

CASE 1 – Wald rule. A) Let's look at set P_2 within which project D_6 is better than D_4 and D_5 – it seems quite astonishing since this project is not the best for none of the considered scenarios! Such a situation may occur when there is not a scenario dominated by others. B) When analysing set P_2 we also observe that the Wald rule discriminates options with even very high outcomes if at least one of their payoffs is lower than the lowest values connected with the remaining variants. C) Moreover other researchers notice that the max-min rule does not distinguish between alternatives with the same security level (see set P_3). However, to overcome that deficiency, the literature offers the **lexicographic max-min (lex-min) rule**

which compares the 2nd-worst outcomes (3rd-worst, 4th-worst etc., if necessary) of the options with the same level w_j and recommends the one with the highest value (Sen, 1984).

CASE 1 – max-max rule. This time we note that in set P_1 and P_3 all the projects obtain the same rank in the ranking, which is completely irrational even for extreme optimists (D_2 is certainly worse than D_1 – only one of its outcomes is positive and the rest is negative!), but it is possible to formulate a lexicographic max-max rule by analogy and avoid such paradoxical situations (then for P_1: $D_1 \gg D_3 \gg D_2$).

CASE 2 – max-max rule. The point which has not been revealed yet by other researchers is the fact that the max-max rule is entirely useless for the optimal spare parts quantity problem. Due to the occurrence of a zero maximal value for each considered order quantity, all the decisions are treated as equivalent and the ranking is worthless.

CASE 3 – Hurwicz rule. A) When comparing securities D_3 and D_4 we note that the last one gets a lower Hurwicz index, although its intermediate rates of returns are much more attractive (6.4 > 0.2, 6.3 > 0, 6.2 > -0.5). B) D_2 and D_3 are allegedly identical due to the same hope and security levels, but it is obvious that D_2 is more favourable to the investor as all its intermediate return rates are close to the hope level! C) The most surprising observation is related to the highest Hurwicz index for security D_1. It has four negative outcomes, but it is supposedly the best since its worst value is slightly higher than the remaining security levels. D) The last point worth emphasizing concerns the relationship between the rankings and the pessimism coefficient – the rankings are the same even for strong pessimists, but in real problems it would be extremely hard to imagine a pessimist investor buying securities D_1!

CASE 4 – Hurwicz rule. In the classical newsvendor problem payoff matrices represent decisions (supplies) which are all Pareto-optimal (there is no alternative dominated by others). Furthermore, each consecutive option is characterized by a growing outcome dispersion and range. A) For $\alpha = 0.5$ all the variants get the same index value, although the first supply is free of risk and the last ones even lead to negative payoffs. The reason of such a phenomenon is as follows – for moderate DMs the procedure just consists in computing the arithmetical average of the hope and security levels. B) Table 4 also shows ranking variations dependent on the pessimism coefficient. They are strongly questionable. We would rather expect a systematic evolution of recommendations for increasing pessimism levels (i.e. $4 \to 3 \to 2 \to 1$). Meanwhile, for $\alpha \in [0.0; 0.49]$ the last supply is the best and for $\alpha \in [0.51; 1.0]$ the first one is suggested. This weak ranking sensitivity does not fit our expectations.

Hence, thanks to the analysis of case 3 and 4, we conclude that the weakness of the Hurwicz rule results from the fact that it does not take into consideration: 1) intermediate values connected with particular alternatives (case 3: D_2 and D_3), 2) the frequency of the occurrence of outcomes equal or close to the hope and security level (case 3: D_1 and D_2), and 3) the diversity of outcome dispersions (case 4) (Gaspars-Wieloch, 2014a).

GENERAL REMARKS. Wald, max-max and Hurwicz rules. Researchers also pay attention to two other aspects. A) In the aforementioned procedures scenarios are treated as conscious opponents who are altering their strategy depending on the outcomes level, which is illogical (Milnor, 1954). B) When the intervals of payoffs for particular events are disjoint, the rules enumerated above compare the alternatives on the basis of one (Wald, max-max) or two (Hurwicz) scenarios only – the rest of events is irrelevant (Puppe, and Schlag, 2009).

CASE 5 – Savage and Hayashi rules. A) The Savage rule seems to be a logical approach. The analysis of case 5 does not lead to any worrisome observations, but it is worth stressing that we can find numerous examples (Gaspars-Wieloch, 2018b) showing that the elimination of a given decision in the payoff matrix may totally reverse the ranking! We do not treat this phenomenon as a drawback – it is just a feature resulting from the evaluation of the positions of particular payoffs in the background of other results from the same scenario. B) However, worrying conclusions can be formulated for the Hayashi rule. According to it all the projects are equivalent. How is it possible? Project D_1, in contrast to other projects, hasn't got any negative profit! Furthermore, the sum of all of its profits is 2, 4 and even 10 times bigger than the sums computed for the remaining projects! The reason why the rankings generated by means of the Hayashi rule may be useless is that all the variants obtain zero index values if for at least one scenario each alternative has got relatively the worst payoff. It is a serious shortcoming of this approach (Gaspars-Wieloch, 2014b).

Bayes rule. We haven't prepared any cases enabling the analysis of the Bayes rule, but its construction is so simple that we can draw conclusions without illustrative examples. A) Contrary to other CDR, this method takes all the outcomes into account, which may be treated as an advantage in the context of the Hurwicz rule evaluation. Nevertheless, if so, as it was mentioned in section 2, the Bayes rule cannot be applied to one-shot decisions. It is a significant limitation since, in connection with a quite uncertain and changing environment, the selected strategy is performed only once in many real decision problems. B) The second deficiency results from the lack of possibility to consider the DM's attitude towards risk. The Bayes rule makes the same recommendations for optimists, moderate people and pessimists! C) Other researchers add that the Bayes criterion can be used provided that symmetry rules for the structure of the scenarios are satisfied and that the available scenarios create an exhaustive set of mutually exclusive events (Hansson, 2011). Hence, possible applications of that procedure are not so large as we could imagine.

5. Summary

The article presents a brief analysis of scenario-based classical decision rules (CDR) in terms of their construction, defects and usefulness in real economic problems. We concentrate on procedures for pure strategy searching which do not refer to the probability calculus. These methods have been already evaluated in the literature, but this work contains many new observations. The general conclusions and suggestions are as follows:

1) The majority of CDR are designed for extreme pessimists. Meantime in real situations the decision makers are rather moderate pessimists or moderate optimists.
2) Recommendations generated by CDR sometimes do not reflect the decision maker's preferences, predictions and expectations – they seem to be irrational. In some specific cases (resulting from the payoff matrix structure) the rankings are illogical or useless.
3) CDR should be applied very carefully, especially when solving real economic decision problems. If there are some alternatives with index values close to the best one, the decision maker ought to take a look at their characteristics. Perhaps they better satisfy his/her needs.
4) Decision making under uncertainty is a field explored by numerous researchers. Some of them are making an attempt to formulate new decision rules, partially based on classical procedures. However, in connection with the fact that CDR are not flawless, it is desirable to construct methods without the defects mentioned in the paper.
5) In order to improve the decision making process on the basis of classical decision rules, it is worth applying additional diverse criteria, at least the standard deviation (Ioan, C., and Ioan, G. 2011).

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