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INVESTMENT RISK ASSESSMENT BASED ON THE LONG-TERM MEMORY PARAMETER

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Purpose: The presence of a long-term memory component in a time series means that even very distant observations exert a certain influence on subsequent implementations of the process. Generally, this relationship is not particularly strong, but it does exist. Interpreting this phenomenon in the context of financial time series, one can come to the conclusion that information that has affected the market some time ago may still be important for the current quotation. The article is devoted to checking the existence of a long-term memory in the financial time series and assessing the investment risk of these series based on the long-term memory parameter.
Design/methodology/approach: In order to study the phenomenon of long-term memory in financial time series, the local Whittle estimator was used, while the investment risk assessment was carried out using the fractal dimension, β-coefficient and standard deviation of rates of return.
Findings: In the first part of the study the author indicated time series which were characterized by the phenomenon of long-term memory. Then, on the basis of selected measures, the risk of investment was estimated and shares with the least risk were indicated.
Research limitations/implications: The results obtained for selected measures showed discrepancies between the shares with the highest and the lowest level of investment risk. Although the results obtained do not give a definite answer which risk measure is more effective, they encourage the use of other measures related to the phenomenon of long-term memory.
Practical implications: Application in portfolio analysis.
Originality/value: The use of the long-term memory parameter to assess the investment risk of shares.

Keywords: phenomenon of long-term memory, local Whittle estimator, financial time series, investment risk.

Category of the paper: Research paper.

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1. Introduction

The issue of the presence of a long-term memory in the series of return rates is inextricably linked to the question about the information efficiency of financial markets and investment risk. In works (Fama, 1970) one may find the statement that the incoming information is immediately incorporated by the market and immediately included in the valuation of assets. Each subsequent change in the state of equilibrium is in this approach an event independent of the previous one. However, reality, especially in economics, is rarely one hundred percent consistent with theoretical postulates. Research has shown that on some markets (periodically) short-term autocorrelations of subsequent rates of return may appear. The question about the presence of long-term relationships, manifested in the form of a long-term memory phenomenon, remains open.

Observation of the phenomenon of long-term memory in the time series means that even very distant observations exert some influence on subsequent realizations of the process. Knowing the precise estimation of the long-term memory parameter, a careful investor would be able to, for example, assess the level of risk and probably thus gain an advantage over other investors.

The aim of the paper is the analysis of occurrence of the phenomenon of long-term memory in selected financial time series using the local Whittle estimator and the assessment of investment risk. The studies used time series created from closing prices of companies listed on the Warsaw Stock Exchange that are part of the WIG20 index, which covered the period from 04.01.2005 to 23.03.2018. The calculations were carried out using the RCran statistical software and the Microsoft Excel package.

2. Long-term memory parameter – Local Whittle Estimator

One of the definitions of the long-term memory phenomenon was proposed by Palma (2007) in his work:

Let: $\gamma(h) = \langle y_t, y_{t+h} \rangle$ be a function of autocovariance for subsequent delays of the h stationary process. This process is characterized by the occurrence of a long-term memory phenomenon, if this function fulfils the condition:

$$\sum_{h=-\infty}^{\infty} |\gamma(h)| = \infty \qquad (1)$$

In the case of short-term memory processes (such as AR (1), $\alpha \in (-1; 1)$), the sum of a similar series is a finite value (Kwiatkowski, 2017). We can also find other definitions of long-term memory (Gurgul, and Wójtowicz, 2006). The stochastic process has a long-term memory if its spectral density function $f(\lambda)$ fulfills the condition:

$$\lim_{\lambda \to 0^+} \frac{f(\lambda)}{c\lambda^{-2d}} = 1 \quad (2)$$

where *c* is a constant, while the *d* parameter is a characteristic of the process memory.

This condition means that for low frequencies close to zero the spectral density of the process goes to infinity. If this condition is met, the autocorrelation function of the process disappears at a hyperbolic rate (Gurgul, and Wójtowicz, 2006):

$$\lim_{h \to \infty} \frac{\rho(h)}{c_\rho h^{2d-1}} = 1 \quad (3)$$

where *c* is a constant.

The parameter *d* defines the character of the process's memory. For *d* > 0, the process has a long-term memory and is called persistent. It shows a tendency to continue trends and to move away from the average value more than a random walk. For parameter *d* = 0, the series is characterized by a lack of memory; an example of a similar process is white noise. For *d* < 0, the spectral density function of the process goes to zero for low frequencies and increases for high frequencies. Such a process is characterized by negative autocorrelation of next implementations, greater variability than random walk and has a tendency to return to the average value (process called anti-persistent (Gurgul, and Wójtowicz, 2006)). These types of time series carry the highest probability of change, and the risk which such a series brings is the greatest.

The long-term memory parameter used in this work was local Whittle estimator, described in Robinson's paper (1995). It is based on the asymptotic properties of the spectral density function $f(\lambda)$ of stationary processes:

$$f(\lambda) \sim G\lambda^{1-2H} \; as \; \lambda \to 0^+ \quad (4)$$

where G is constant that satisfies the condition $G \in (0, \infty)$ and H is a long-term memory parameter, such that $H \in (0, 1)$. For low frequencies and $H \in (0, \frac{1}{2})$ the value of function $f(\lambda)$ tends to zero, whereas while $H \in (\frac{1}{2}, 1)$ the value of spectral density function tends to infinity. If $H = \frac{1}{2}$ function $f(\lambda)$ tends to a finite positive constant at zero frequency.

Local Whittle estimator is based on the periodogram $I(\lambda)$ of the time series $\{y_t\}$:

$$I(\lambda) = \frac{1}{2\pi n} \left| \sum_{j=1}^{n} y_j \, e^{i\lambda j} \right|^2 \quad (5)$$

where n is the length of the time series, m is a certain positive integer, satisfying the condition $m < n / 2$, while λ_j is frequencies, such that:

$$\lambda_j = \frac{2\pi j}{n}, \text{ for } j = 1, 2, \dots, m \tag{6}$$

To calculate H we have to find the minimum of objective function:

$$Q(G, H) = \frac{1}{m}\sum_{j=1}^{m}\left\{logG\lambda_j^{1-2H} + \frac{\lambda_j^{2H-1}}{G}I_j\right\} \tag{7}$$

where $I_j = I(\lambda_j)$, and m is integer satisfying $m < n$. If we replace G variable with estimator $\hat{G}$:

$$\hat{G}(H) = \frac{1}{m}\sum_1^m \lambda_j^{2H-1}I_j \tag{8}$$

then the objective function (7) may be simplified to the function of one variable:

$$R(H) = log\,\hat{G}(H) - (2H - 1)\frac{1}{m}\sum_1^m log\lambda_j \tag{9}$$

Local Whittle estimator belongs to class of semiparametric estimators. Its main advantage is that it doesn't require a model specification. If we choose the parameter m appropriately, the estimation result is not biased even in case of presence of short-term memory components, such as autoregression or moving average phenomenon. What is more, local Whittle estimator allows us to avoid assumption of the Gaussianity of the studied time series, unlike many other estimators. Under conditions (Robinson, 1995), such as:

$$\frac{1}{m} + \frac{m}{n} \to 0 \text{ as } n \to \infty \tag{10}$$

local Whittle estimator is consistent and statistic

$$\sqrt{m}(\hat{H} - H_0) \tag{11}$$

is normally distributed $N(0, \frac{1}{4})$. Taqqu and Teverovsky (1997) shown, that when the time series are long enough and we choose appropriate value of parameter m, the local Whittle estimator performs not much worse than the original Whittle method with correct model specification.

3. Measure of investment risk

Investment risk can be defined as the possibility of an effect inconsistent with expectations or in a narrower sense - the possibility of loss (Jajuga, K., and Jajuga, T., 2006). In the case of the shares market, the risk of a security paper is often equated with the volatility of a series of its prices or rates of return (Orzeszko, 2010). The most popular measure of variation is standard deviation (variance). The more risky shares have a larger standard deviation of a series of return rates. The reasoning presented justifies the use of a local Whittle estimator as an alternative measure of volatility, and therefore the risk of investing in securities.

The fractal dimension is also one measure of variation. It determines the degree of raggedness of the time series chart, which allows to assume that the larger the dimension of series, the greater its variability. In this case, securities whose ranks of returns have a larger dimension are more variable, and that means they are more risky (Zeug-Żebro, and Miśkiewicz-Nawrocka, 2018).

The fractal dimension of a given geometric object A can be calculated by estimating the minimum number of closed hypercubes with side length ε needed to cover it. This dimension is determined based on the following formula:

$$D(A) = \lim_{\varepsilon \to \infty} \frac{\ln L(A,\varepsilon)}{\ln\left(\frac{1}{\varepsilon}\right)} \tag{12}$$

where $L(A,\varepsilon)$ is the minimum number of hypercubes with side length ε.

One of the procedures allowing the calculation of the fractal dimension $D(N)$ of the time series is the analysis of the rescaled range or, briefly, the R/S analysis (Hurst, 1951). This method first of all involves estimating the value of the Hurst exponent H^* (Chun, et al., 2002) and then determining the fractal dimension according to the formula (Zwolankowska, 2000):

$$D(N) = 2 - H^* \tag{13}$$

Another group of risk measures are measures of sensitivity that inform about the impact of various risk factors on securities prices. Thus, the greater the sensitivity of the price of securities to the changes in the factors determining it, the greater their risk. The known and commonly used measure of sensitivity is the βeta coefficient, which is one of the measures of market risk (Ostrowska, 2007).

The βeta coefficient called the shares aggressiveness coefficient is a measure of the sensitivity of income from a given share to the statistical volatility of the entire securities market, i.e. it is a sensitivity measure of this share in relation to the shares of average risk. βeta describes the activity of a security's returns responding to swings in the market. The coefficient βeta of *i*-th shares (β_i), being a relative measure of market risk, is determined based on the formula:

$$\beta_i = \frac{\sum_{i=1}^{n} \left(R_{it} - \overline{R}_i\right) \cdot \left(R_{Mt} - \overline{R}_M\right)}{\sum_{t=1}^{n} \left(R_{Mt} - \overline{R}_M\right)^2} \tag{14}$$

where n is the number of periods from which observations of the rates of return are derived, R_{it} is the rate of return of the *i*-th share in the *t*-th period, R_{Mt} is the rate of return of the market index in the t-th period, $\overline{R}_i$ is the arithmetic mean of the returns of the i-th share, $\overline{R}_M$ is the arithmetic average of the return of the market index (Ostrowska, 2007).

The βeta calculation is used to help investors understand whether a stock moves in the same direction as the rest of the market, and how volatile or risky it is compared to the market.

The βeta values greater than 1 mean more risky shares than the market (hence they should bring a correspondingly greater return), while values lower than 1 and higher than zero are shares less risky than the market (and therefore less profitable). βeta less than zero is almost unheard of on the stock market and points to securities moving against the market.

4. Empirical analysis of proposed measures of investment risk

In the study we used the financial time series which were the shares price of selected companies listed on the Warsaw Stock Exchange: AGORA, AMICA, BZWBK, CCC, CDPROJEKT, POLNORD, DEBICA, FORTE, GROCLIN, INGBSK, KGHM, RELPOL, PEKAO, PKNORLEN and PKOBP. Each of the time series included 3312 observations for the period of time from 04.01.2005 to 23.03.2018. Chosen time window lets us consider market condition both in financial crisis of 2007 and in periods of stable growth.

According to Taqqu and Teverovsky (1997) suggestion, in the estimation of the parameter H we chose value of m parameter as integer part of $\frac{N}{32}$, where N denotes length of the considered time series. Estimation results are shown in the Table 1. Second column presents the estimated value of parameter H for m = 103 (Zeug-Żebro, and Szafraniec, 2018) . The third one shows the estimated standard deviation of the estimator, while fourth column contains the test statistic with formula:

$$\frac{(\hat{H}-H)}{\sqrt{\frac{1}{4m}}} \tag{15}$$

where as the testing value of parameter H we took H = 0.5 (characteristic for white noise). Fifth column shows the p-value for two-tailed test.

Results in table 1 show that in the considered period companies listed on the Warsaw Stock Exchange were characterized by a heterogenous parameter of long-term memory. For seven companies estimated values of H were greater than 0.5 and results were statistically significant. The highest value of H, equal to about 0.661, was estimated for RELPOL company. It is worth noting that no statistically significant value of the H parameter was less than 0.5, i.e. it did not point to the phenomenon of anti-persistence.

The results provide evidence of presence of long-term memory in stock returns for some financial instruments listed on Warsaw Stock Exchange. However, the strength of such dependence isn't as significant as in case of some naturally occurring time series.

Table 1.
Local Whittle Estimator: estimation results

Company	Estimated *H* value	Standard deviation of estimator	Test statistic	*p*-value
AGORA	0.513221	0.049266	0.268361	0.788000
AMICA	0.636890	0.049266	2.778573	**0.005100**
BZWBK	0.524601	0.049266	0.499345	0.618000
CCC	0.552583	0.049266	1.067322	0.286000
CDPROJEKT	0.548872	0.049266	0.991984	0.321000
POLNORD	0.625426	0.049266	2.545870	**0.011010**
DEBICA	0.576986	0.049266	1.562647	0.118000
FORTE	0.590570	0.049266	1.838371	**0.066020**
GROCLIN	0.615625	0.049266	2.346933	**0.019011**
INGBSK	0.611068	0.049266	2.254436	**0.024001**
KGHM	0.594964	0.049266	1.927558	**0.054002**
RELPOL	0.661401	0.049266	3.276082	**0.001001**
PEKAO	0.429549	0.049266	-1.430001	0.153000
PKNORLEN	0.471148	0.049266	-0.585633	0.558000
PKOBP	0.512744	0.049266	0.258670	0.796000

The companies in which the estimated *H* values were statistically significant were marked in bold.

According to the (presented in the theoretical part of the article) interpretation of the local Whittle estimator, the *H* value can be treated as a measure of investment risk. Below is the ranking of companies (Figure 1) from the least risky to those with the highest risk. Only those companies for which the value of the local Whittle estimator was statistically significant were taken into account.

Estimated *H* value

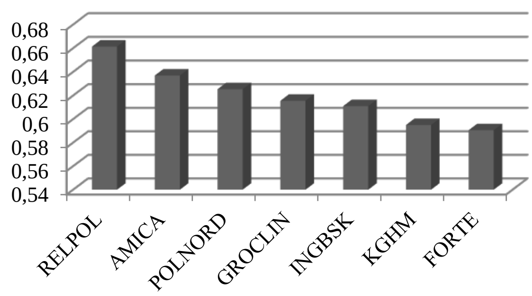

Figure 1. Ranking of companies created according to the value of the local Whittle estimator.

Based on the ranking presented in Figure 1, regarding the value of the local Whittle estimator, the companies that were characterized by the lowest risk were RELPOL and AMICA. However, the highest risk was recorded for KGHM and FORTE.

In the next step of the research, the fractal dimension, the βeta coefficient and the standard deviation of the return rates of the studied time series were estimated. The values obtained are shown in Table 2.

Table 2.
Results of estimating the fractal dimension, the βeta coefficient and standard deviation for the time series of companies included in the WIG20 index

Company	Standard deviation	βeta coefficient (WIG)	Fractal dimension
AGORA	0.023338	0.808627	1.433823
AMICA	0.023431	0.749520	1.380541
BZWBK	0.021694	1.171308	1.452921
CCC	0.021271	0.632129	1.417602
CDPROJEKT	0.036585	1.054484	1.443720
POLNORD	0.033944	1.090789	1.393031
DEBICA	0.017568	0.458939	1.409980
FORTE	0.021810	0.475318	1.388702
GROCLIN	0.035003	1.056416	1.397703
INGBSK	0.018879	0.799925	1.420301
KGHM	0.027297	1.547284	1.403130
RELPOL	0.037667	0.684825	1.430541
PEKAO	0.022013	1.393459	1.498613
PKNORLEN	0.021912	1.256610	1.453121
PKOBP	0.020219	1.307408	1.480310

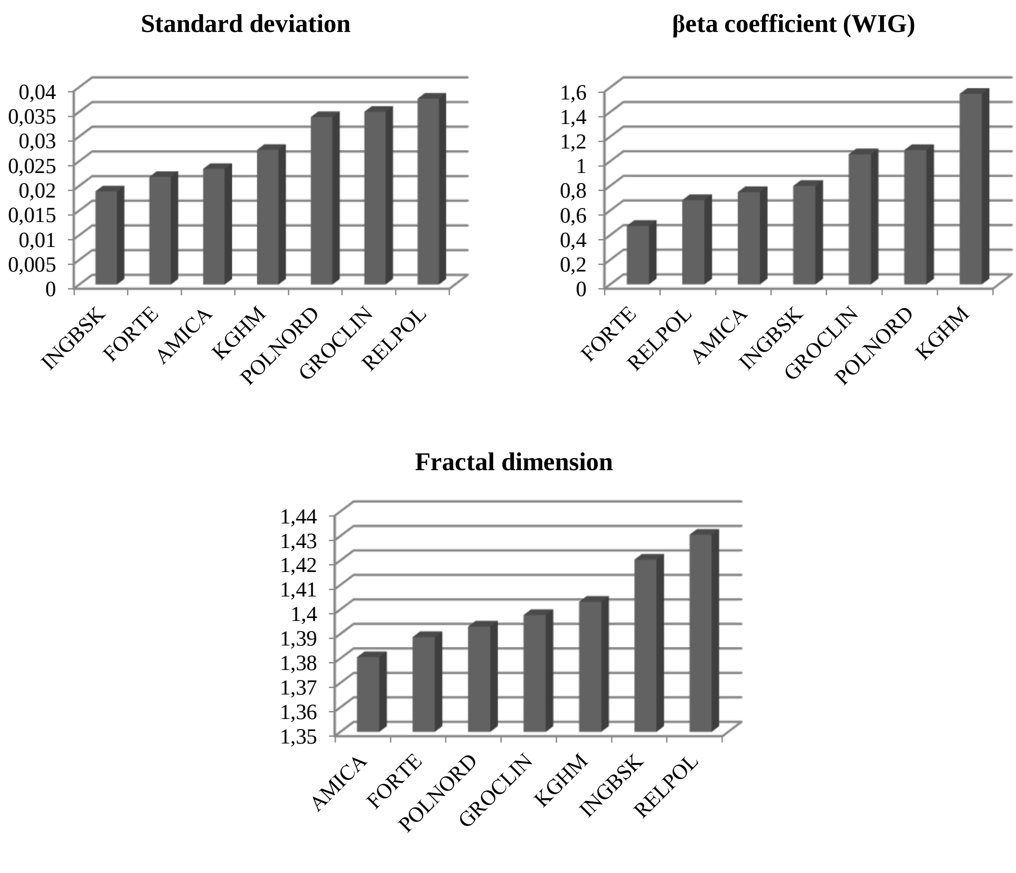

Figure 2. Ranking of companies created according to the value of designated risk measures.

The obtained rankings[1] show that a completely different ordering according to the risk measure value was obtained using the local Whittle estimator, the fractal dimension, the βeta coefficient and standard deviation. It is worth paying attention to the shares that were characterized by the lowest risk level. According to the βeta value, these were: FORTE and RELPOL; for the fractal dimension: AMICA and FORTE; and finally for standard deviation: INGBSK and FORTE. The value of risk calculated based on the standard deviation and the fractal dimension obtained for the RELPOL time series is surprising, according to which the investment in this company is the most risky. Meanwhile, as indicated earlier, when using the local Whittle estimator as a risk measure, this company belongs to the group of the least risky investments. The reverse situation is observed in the case of FORTE. Interestingly, according to the values obtained for the standard deviation and the βeta coefficient, the investment in FORTE is the least risky, and according the level of risk obtained for the local Whittle estimator is the most risky.

5. Conclusions

The first part of the study involved an analysis of the phenomenon of long-term memory of selected financial time series. The conducted analysis indicates that a part of stock exchange stocks is characterized by a lack of long-term memory. The null hypothesis was rejected only in the case of 7 shares listed on the Warsaw Stock Exchange. This means that there is a relationship in these ranks between observations, even considerably distant in time. No anti-persistent phenomenon was observed during the study.

In the second part of the study the author conducted a risk analysis based on the local Whittle estimator, the fractal dimension, the βeta coefficient and standard deviation. From the rankings obtained during the study of the level of risk of financial time series, it follows that a different ordering was obtained using these measure. Some compliance was observed only for two companies – FORTE and RELPOL. Their positions in the rankings, however, are opposed (due to the considered risk measure). The results obtained for some measures indicated the first places in the ranking (for these companies), the last ones for other measures. Although the results obtained do not give a definite answer on which risk measure is more effective, they encourage further research in this area.

[1] Only those companies for which the value of the local Whittle estimator was statistically significant were taken into account.

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