

## THE MULTI-CRITERIA FUZZY APPROACH TO BUILDING AND EVALUATING EFFECTIVE PORTFOLIOS

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**Purpose:** The aim of the article is to compare the results of effective portfolios obtained after the initial selection using multi-criteria methods with the results of the market portfolio.

**Design/methodology/approach:** When selecting a long-term portfolio, a fundamental analysis can be used to assess a company's economic and financial condition. This analysis is based on fundamental and market indicators. By treating selected indicators as evaluation criteria, the problem can be considered as a multi-criteria problem. In the analyses the TOPSIS methods were used (standard and fuzzy one), which enabled the approach to the issue in a non-standard way.

**Findings:** Three effective portfolios were determined: two of them were obtained after the initial selection of companies using selected multi-criteria methods, the third was generated from the set of all considered companies. The results of these portfolios, estimated for the whole of 2018, were compared with the market portfolio represented by the WIG20 index. The analysis showed that including the fuzzy approach when selecting a portfolio, it is possible to construct more profitable portfolios compared to the market portfolio.

**Research limitations/implications:** The problem requires further research to confirm the recommendations made.

**Practical implications:** Using the proposed approach, we can methodically build more profitable portfolios than the market portfolio.

**Originality/value:** The values of criterion assessments from selected years were treated as triangular fuzzy numbers – this enabled the use of fuzzy approach and the selection of portfolios more attractive than the market one. The study may be of interest to stock market investors.

**Keywords:** Fuzzy TOPSIS, multi-criteria problem, effective portfolios, market portfolio.

**Category of the paper:** Research paper.

### 1. Introduction

Constructing portfolios is a challenge for any potential investor. There are many ways to support this issue. In the process of selecting a long-term portfolio, you can, for example, involve elements of fundamental analysis and use fundamental and market indicators in this

problem. The assessment based on them will enable the selection of enterprises with a stable economic and financial situation – the stability of the company inspires confidence, makes it a more reliable investment object. Taking into consideration the methods of fundamental analysis raises the question about the time range from which to draw data – too long period can obscure the current situation of the company, while too short may not reflect the actual state of affairs. The literature proposes that this period should cover from three to the last five years (Tarczyński, 2002). Every decision maker would also like to invest in companies that generate high profit with low risk, which usually does not go hand in hand. In this situation, a solution is to determine effective portfolios – portfolios with the lowest risk are generated at the expected rate of return or the average rate of return is maximized at a given risk level (Jajuga, and Jajuga, 2015). The article combines the determination of effective portfolios with pre-selection carried out using a multi-criteria method that takes into account the fuzzy approach. The results of the portfolios obtained are compared to the results of the market portfolio.

The aim of the article is to compare the results of the market portfolio with the effective portfolios determined after the initial selection using multi-criteria methods and the recommendation of a fuzzy approach to grouping companies constituting the basis for portfolio selection. The research hypothesis is that by involving in the issue the chosen multi-criteria method in a fuzzy approach, more profitable portfolios are obtained than the market portfolio.

## 2. Research methodology

The evaluation of objects through the prism of selected criteria makes it possible to use multi-criteria methods in research. The analysis of literature and previous considerations showed the legitimacy of using the TOPSIS methods to support portfolio selection (Chen, and Hung, 2009; Ece, and Uludag, 2017; Kazemi et al., 2014; Liu, et al., 2012; Nguyen, and Gordon-Brown, 2012; Raei, and Bahrani Jahromi, 2012; Pośpiech, 2017a, 2017b, 2018; Pośpiech, and Mastalerz-Kodzis, 2015, 2016). There are several versions of the TOPSIS method (standard, fuzzy, interval, with linguistic data), the use of which depends on the form of the issue formulated. These methods allow you to construct the ranking of objects.

In the considerations, as the company's evaluation criteria, selected fundamental and market indicators were adopted, while the applied methods are the TOPSIS method in standard approach (criterion evaluations are crisp values) and fuzzy TOPSIS method – FTOPSIS (criterion evaluations are triangular fuzzy numbers whose parameters are successively pessimistic, expected and optimistic assessment of the decision variant). The procedure for building the multi-criteria ranking takes place in stages, which in the case of selected methods includes: determining the appropriate decision matrix, building a weighted normalized decision matrix, determining the ideal and anti-ideal solutions, calculating the distance of decision

variants from the given solutions and calculating the relative distance  $S_i$  for each variant  $i$  on the basis of which the ranking is constructed (the higher the value, the higher the ranking). A detailed description of the methods can be found in many studies, including: (Hwang, and Yoon, 1981; Jahanshahloo, et al., 2006; Lai, et al., 1994; Roszkowska, and Wachowicz, 2013; Trzaskalik, 2014). It was found that for the fundamental and market indicators, data from three years will be taken into account: for the case of the standard TOPSIS method, criterion evaluations will be expressed using one value (average of the analysed three years), while in the fuzzy approach, criterion assessments from three years will be treated as appropriate parameters of triangular fuzzy numbers:

$$\tilde{a}_i^{(k)} = (l_{ik}, m_{ik}, u_{ik}) \quad (1)$$

where:

$k$  – specifies the criterion number,

$l_k$  – the smallest value of the criterion assessment for the company from three years,

$m_k$  – the middle value of the criterion assessment for the company from three years,

$u_k$  – the highest value of the criterion assessment for the company from three years.

The following fundamental and market indicators were adopted as the company assessment criteria (Leszczyński, 2004; Tarczyński, 2001, 2002; Trzaskalik, 2006; Tyran, 2001):

- return of assets ROA (net income/average total assets),
- return of equity ROE (net income/shareholder equity),
- P/BV (price-book value),
- P/E (price-earnings ratio).

The research covered WIG20 index companies that were included in it in December 2017. Data for fundamental and market indicators were taken from the Financial Statements of the surveyed companies from 2015-2017, while the rates of return came from 2017 and 2018. In determining the effective portfolios, average rates of return from 2017 were taken into account. In ordering the companies the TOPSIS method in the standard approach (variant I) and in the fuzzy approach (variant II) were used. All the criteria were treated as just as important, and each criterion was maximized. Effective portfolios were determined using the classic Markowitz approach (Markowitz, 1952) – the portfolio shares were obtained by minimizing the variance of the portfolio at a given level of expected rate of return and standard terms for shares (non-negative and the sum equals to one).

### 3. Main results of empirical analysis

The first stage of the analyses was to build a ranking of companies. The ordering resulting from the application of multi-criteria procedures is given in Table 1.

**Table 1.**

*Values of the  $S_i$  indicator and rankings according to the individual variants*

Company	Variant I (TOP)		Variant II (FTOP)	
	$S_i$	Ranking	$S_i$	Ranking
ALR	0.577	15	0.503	14
ACP	0.607	11	0.506	12
BZW	0.627	9	0.531	8
CCC	0.968	1	0.792	1
CPS	0.644	6	0.538	6
ENG	0.600	14	0.497	15
EUR	0.724	3	0.586	4
JSW	0.473	18	0.381	18
KGH	0.416	19	0.290	20
LTS	0.630	8	0.506	13
LPP	0.856	2	0.686	2
MBK	0.602	13	0.509	11
OPL	0.476	17	0.377	19
PEO	0.619	10	0.523	9
PGE	0.576	16	0.443	16
PGN	0.643	7	0.534	7
PKN	0.723	4	0.617	3
PKO	0.604	12	0.510	10
PZU	0.663	5	0.574	5
TPE	0.241	20	0.417	17

Adapted from: "The Risk of Multi-Criteria Portfolios Taking into Account the Fuzzy Approach" by E. Pośpiech. 2018.

On the basis of the obtained rankings, subsets of companies were selected which constituted the basis for selecting the portfolio. As a result of the analyses,  $S_i$  was assumed at the level of 0.5 as the allocation threshold to the group from which the portfolios were built. The grouping result is shown in Table 2.

**Table 2.**

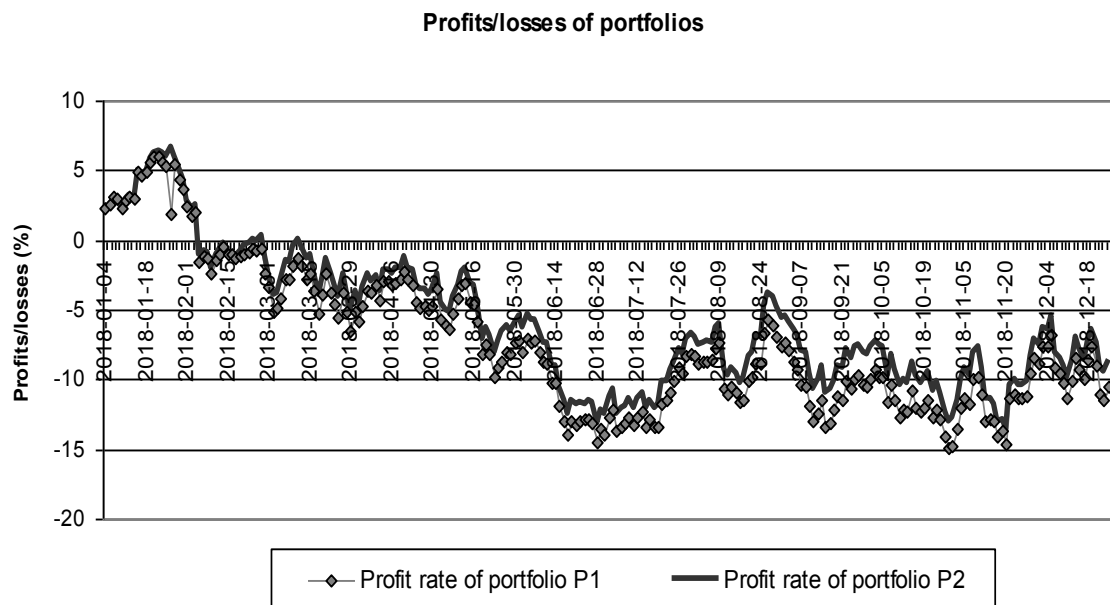
*The sets of companies constituting the basis for portfolio selection*

Variant – Number of companies	Groups of companies
I (TOP) – 16	ALR, ACP, BZW, CCC, CPS, ENG, EUR, LTS, LPP, MBK, PEO, PGE, PGN, PKN, PKO, PZU
II (FTOP) – 14	ALR, ACP, BZW, CCC, CPS, EUR, LTS, LPP, MBK, PEO, PGN, PKN, PKO, PZU

Adapted from: "The Risk of Multi-Criteria Portfolios Taking into Account the Fuzzy Approach" by E. Pośpiech. 2018.

Solving the optimization task, in which the variance of the portfolio was minimized at a given level of expected rate of return (average positive return rates of the considered companies), the structure of effective portfolios was obtained. Based on this structure, hypothetical portfolios were built on 03.01.2018. Their assessments were made on the basis of

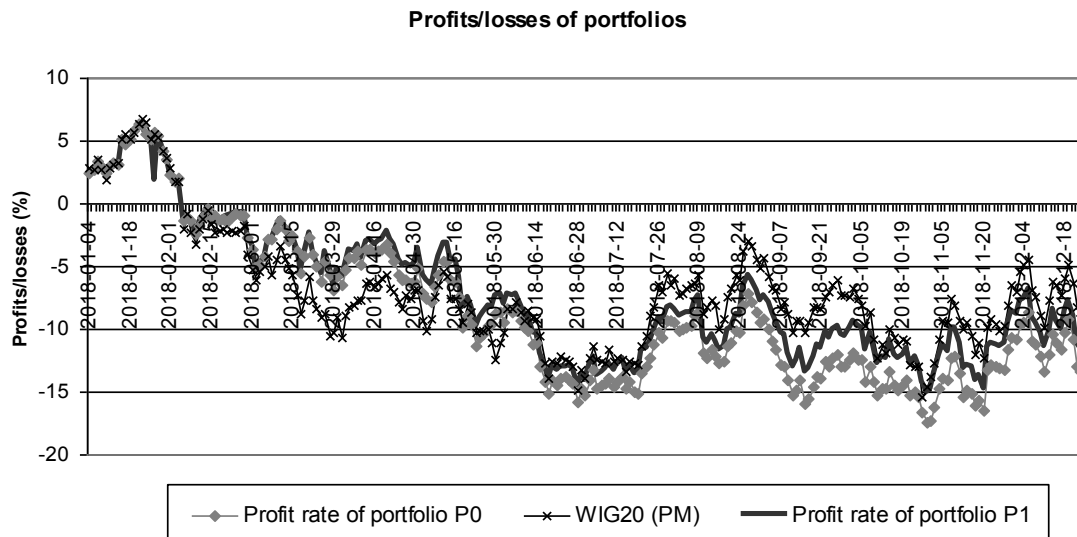
the profits and losses they would have recorded throughout 2018. Figure 1 shows the profits/losses of effective portfolios obtained after the initial selection using the TOPSIS methods – P1 denotes the portfolio generated after the selection using the standard method, and P2 – the portfolio after the initial fuzzy selection.



**Figure 1.** Profits/losses of P1 and P2 portfolios. Source: Own study based on the data from the websites: [www.bankier.pl](http://www.bankier.pl), [www.gpw.pl](http://www.gpw.pl), <http://infostrefa.com/infostrefa/pl/archiwum>.

For generated portfolios, profits were recorded only in the initial period (January and beginning of February). During this time, the results of the portfolios received were similar. After this period, portfolios started to record losses, and there was also a larger discrepancy of results between P1 and P2 portfolios – P2's portfolio generated lower losses. In general, the portfolio obtained after the initial fuzzy selection records better results for almost the entire considered period.

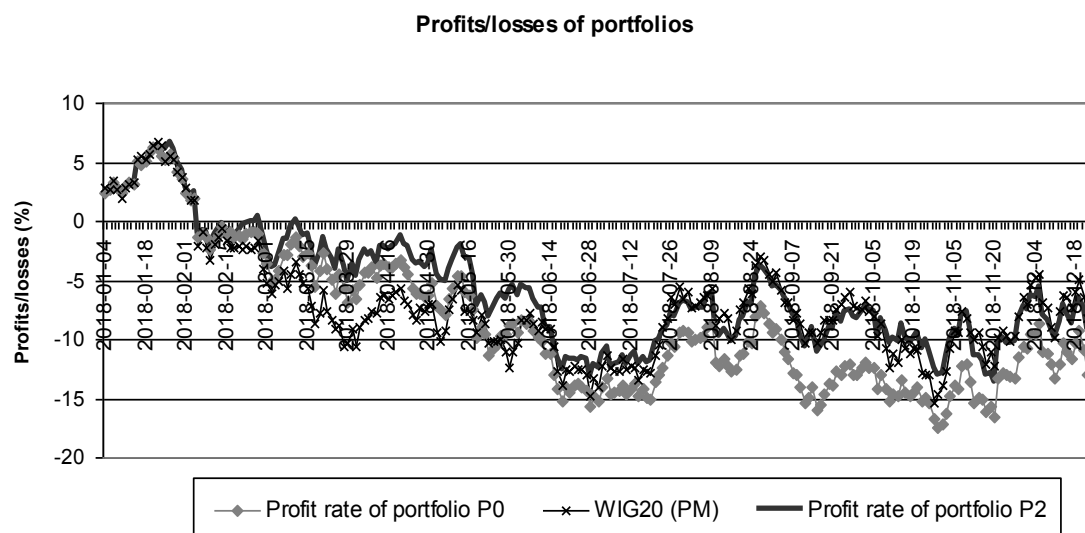
The results of these portfolios were compared with the results of two others: the effective portfolio designated on the basis of all twenty companies surveyed (without pre-selection) – it was marked P0, and the market portfolio (the WIG20 index was taken into account) – designated as PM. The situation is presented in figures 2 and 3.



**Figure 2.** Profits/losses of P0, P1 and PM portfolios. Source: Own study based on the data from the websites: [www.bankier.pl](http://www.bankier.pl), [www.gpw.pl](http://www.gpw.pl), <http://infostrefa.com/infostrefa/pl/archiwum>.

Analysis of the presented results allows to conclude that the P0 portfolio (effective, without preliminary selection of companies) gives comparable or worse results than the portfolio selected after the initial selection with the multi-criteria method of TOPSIS – the selection is therefore justified. In comparison with P1, the PM market portfolio has similar results in the initial period and worse results until the beginning of June. After this time, the market portfolio is achieving better results (it has lower losses). The conclusion can be drawn that the effective portfolio obtained after the initial selection with the multi-criteria method of TOPSIS gives not worse results for the first few months after its construction.

Figure 3 shows the results of P0, PM and P2 portfolios.

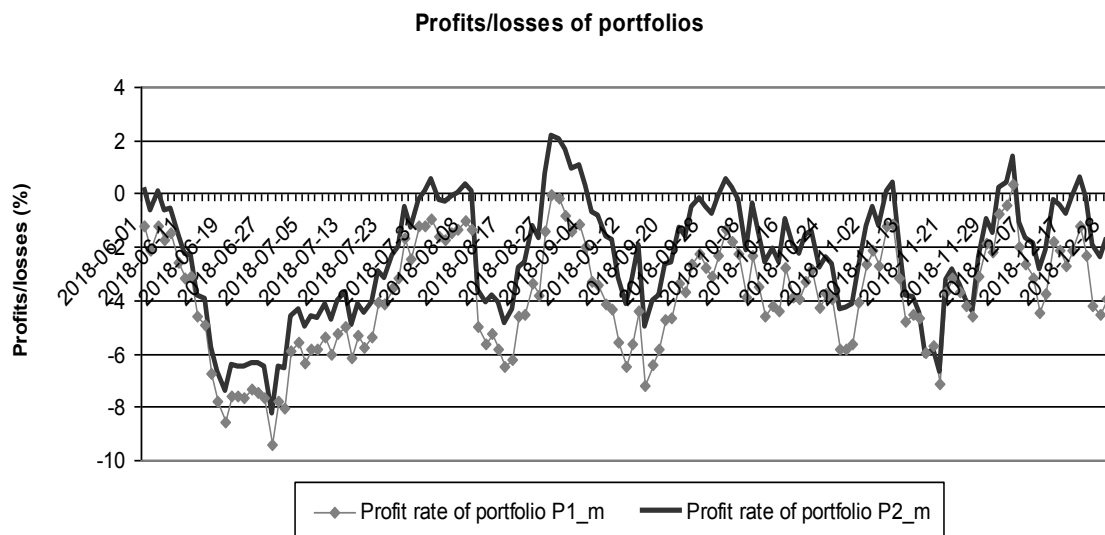


**Figure 3.** Profits/losses of P0, P2 and PM portfolios. Source: Own study based on the data from the websites: [www.bankier.pl](http://www.bankier.pl), [www.gpw.pl](http://www.gpw.pl), <http://infostrefa.com/infostrefa/pl/archiwum>.

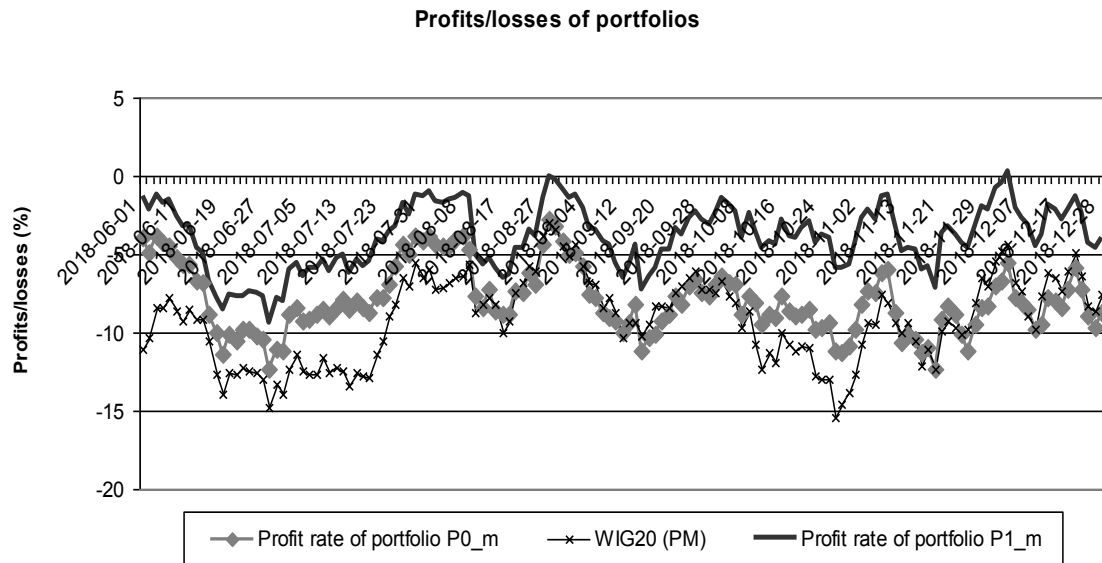
In the initial period, the P2 portfolio records comparable results with the market portfolio, after which for several months it is characterized by lower losses. The situation changes at the end of July, the unambiguous domination of one of the portfolios is not visible: in some periods P2 has smaller losses, in some periods the PM slightly dominates over P2.

#### 4. Discussion

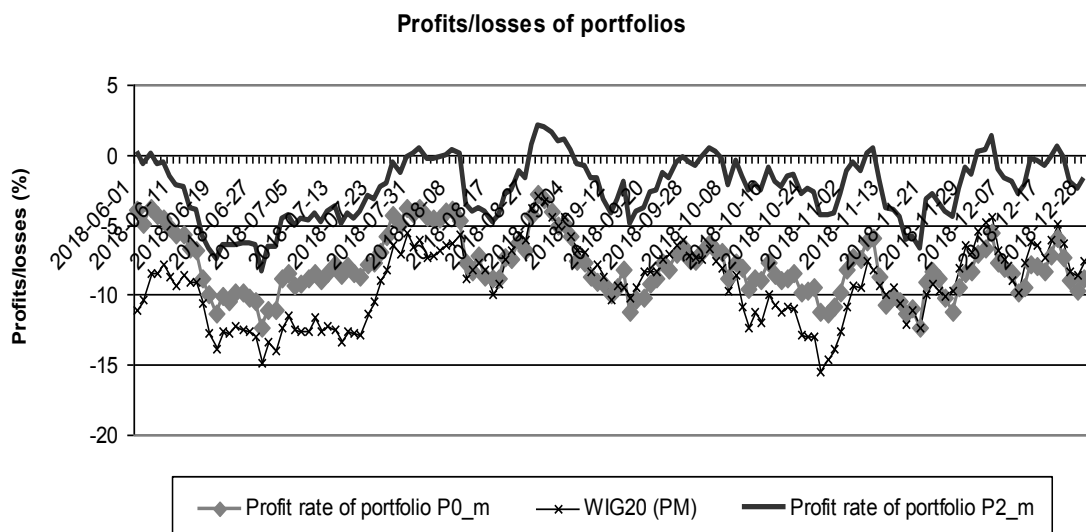
Comparing all four examined portfolios, it can be noticed that for over a month, these portfolios achieve comparable results, after which the domination of the P2 portfolio is visible for several months. The situation changes in June and July – from this moment the results are ambiguous or the market portfolio has better results. It may suggest that after a period of unambiguous domination of portfolios generated after the initial selection using the TOPSIS or FTOPSIS method, the structure of portfolios should be modified. Therefore, such a modification was made, accepting 01.06.2018 as the day of change. The average rates of return from the period 01.06.2017-30.05.2018 were used to determine effective portfolios. The profits of the modified portfolios for the period 01.06.2018-28.12.2018 were estimated; new portfolios were marked as before, attaching "\_m" to each of them. The results are presented in Figures 4-6.



**Figure 4.** Profits/losses of modified P1\_m and P2\_m portfolios. Source: Own study based on the data from the websites: [www.bankier.pl](http://www.bankier.pl), [www.gpw.pl](http://www.gpw.pl), <http://infostrefa.com/infostrefa/pl/archiwum>.



**Figure 5.** Profits/losses of modified P0\_m, P1\_m and PM portfolios. Source: Own study based on the data from the websites: [www.bankier.pl](http://www.bankier.pl), [www.gpw.pl](http://www.gpw.pl), <http://infostrefa.com/infostrefa/pl/archiwum>.



**Figure 6.** Profits/losses of modified P0\_m, P2\_m and PM portfolios. Source: Own study based on the data from the websites: [www.bankier.pl](http://www.bankier.pl), [www.gpw.pl](http://www.gpw.pl), <http://infostrefa.com/infostrefa/pl/archiwum>.

A comparison of the modified effective portfolios obtained after the prior selection of the companies makes it possible to state that the P2\_m portfolio (selection using FTOPSIS method) achieves better results than the P1\_m portfolio (selection using TOPSIS method) over the entire period considered. The modified P0\_m portfolio (without selection of companies) mostly achieves better results than the market portfolio (over 2/3 of the period). The P1\_m portfolio has definitely better results than the P0\_m portfolio and the market portfolio, and thus the P2\_m portfolio achieves the best results (the smallest losses and even small profits were recorded). The portfolio composition modification made it possible to obtain more profitable portfolios than the market portfolio.



## 5. Summary

The determination of effective portfolios in the proposed approach enabled the construction of portfolios that are more profitable than the market portfolio. Portfolio evaluation was made on the basis of potential profits set for the following days of 2018. At the beginning of the period under consideration, all portfolios achieved similar results, and for several months (at least until the end of May) the following relation is visible:  $P2 \succ P1 \succ P0 \succ PM$ , where the character „ $\succ$ ” is a symbol of the preference relation. In the second part of the year, the above preferences were distorted and the market portfolio dominated in terms of profits (losses). Modification of effective portfolios, which was made on 01.06.2018, caused that for over 2/3 of the analysed period there was still a relation:  $P2 \succ P1 \succ P0 \succ PM$ , and only for about 1/3 of this period the preference was as follows:  $P2 \succ P1 \succ PM \succ P0$ .

The conducted analyses showed that thanks to the initial selection of companies (constituting the basis for choosing the portfolio) using multi-criteria TOPSIS methods, it is possible to determine effective portfolios with results not worse than the market portfolio. At the same time, the thesis that the inclusion of the fuzzy method allows you to generate the best-performing portfolios was confirmed. At this stage of research, it can therefore be concluded that the research goal has been achieved and the hypothesis has been positively verified.

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